MATHEMATICS

(CLASS VI)

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CLASS VI

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FOREWORD

The West Bengal Board of Secondary Education,has taken up the responsibility of publishing the Mathematics book for Class VI as per Govt. order No. 968/1(1)-S.E. (Primary) dt. 31. 10. 94. We have not changed the book which is written simply and realistically. The syllabus has also remained unchanged. Only a few errors have been corrected with carefulness. Due to price-hike paper and printing have become costly. But in the interest of the students and guardians the price of the book has been kept as low as possible, though against heavy odds.

Finally, the Board will welcome all constructive criticism for the betterment of the book in future. The Board's effort will be considered successful if the book really benefits the learners and enhances their interest in the subject.

4 April, 1995
77/2, Park Street
Calcutta-700 016

Chittaranjan Bandopadhyay
President
West Bengal Board of
Secondary Education

PREFACE

With the aim of consolidating education among general mass of students, West Bengal Government has taken over the responsibility of printing and publishing of the book on Mathematics meant for students of Class VI of West Bengal. A few words are being placed here for the teachers who will impart instruction on the contents of the book. Of the Mathematical operational symbols which have been established gradually, elementary symbols and their extensive uses, computations associated with them in different spheres and a little of geometry with the help of ruler and compass are included in it. Inverse of a number and their multiples form the basis theme of the book. If the teachers undertake the effort to explain the inverse operation with parts of geometrical figures, students will understand the principles underlying the operations. As a result they will form a good ground for further development of mathematical analysis. So, the teachers should remain aware about taking help of models to explain the steps to students as far as possible. Education Directorate will remain grateful to teachers if the book be found upto expectation. It will be available free of cost to the students of Class VI of Schools of West Bengal.

Writers' Buildings
Calcutta
The 28th May, 1981

Dr. S. C. Das Gupta
Director of Public Instruction
West Bengal

FOREWORD

The preparation of this textbook in Mathematics, prescribed for Class VI from 1981, is the first venture on the part of the Board in a non-language subject, involving the combined and long-sustained labour of experienced practising teachers of secondary schools and training colleges and the expertise of reputed university professors. It has to be unreservedly owned that without their active help, unfailing guidance and constant encouragement, the Board would not have taken the momentous decision to prepare this textbook. No less inspiring was the wholehearted approval of the West Bengal Government. Further, it was a moment of supreme pleasure for the Board when the Government undertook the responsibility of publishing this textbook in Bengali, Hindi, Napali and English and distributing copies thereof to Class VI students free of cost. This is perfectly of a piece with the Government's declared policy of having free, universal primary and secondary education, a unique landmark in the history of education in the state.

The easy methods of working out sums, free from the traditional methods and grooves of thinking, constitute a salient feature of this textbook. It is an attempt to teach Mathematics by intelligible means in a language akin to the children's milieu and life-experience. It is also an approach to arouse and sustain their interest in the subject, for long the bogy of young minds.

Students in lower classes come from all sections, the well-to-do and the poor, the latter having been long denied the knowledge of the three R's by adverse circumstances. This textbook has, therefore, been prepared keeping in view the unequal levels of mental development of children coming from these two sections of society.

Though this first endeavour of the Board may not be above reproach, yet it is felt that it has not failed to project its significance in terms of social consciousness and benefit.

However, the Board will feel grateful if teachers point out the flaws, if any, in the work and offer constructive suggestions for its improvement.

B. Moitra
President
West Bengal Board of Secondary Education

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ARITHMETIC

CHAPTER 1

Revision of previous work

Metric System

Indian Currency

100 Paise = 1 Rupee

Measure of Length

10 Millimetres=1 Centimetre (cm)
10 Centimetres=1 Decimetre (dm)
10 Decimetres=1 Metre (m)
10 Metres=1 Decametre (Dm)
10 Decametres=1 Hectometre
10 Hectometres=1 Kilometre (Km)

1 Metre=100 Centimetres, 1 Kilometre=1000 metres

Measure of Area

100 Square millimetres (sq mm)=1 Square Centimetre (sq.cm)
100 Square centimetres=1 Square decimetre (sq.dm,
100 Square decimetres=1 Square metre (sq.m)
100 Square metres=1 Square decametre (sq.Dm) or 1 are
100 ares or 100 Square decametres= 1 Hectare
1 are=100 sq.m.; 1 Hectare= 10000 sq.m.

Measure of Volume

1000 Cubic millimetres=1 Cubic centimetre (c.c)
1000 Cubic centimetres= 1 Cubic decimetre
1000 Cubic decimetres= 1 Cubic metre

Measurement of Capacity of liquids.

10 millilitre = 1 centilitre

10 centilitre = 1 decilitre

10 decilitre = 1 litre

Also 1 litre = 1000 millilitres = 1 Cubic decimetre

1 litre = 1000 Cubic Centimetres (c.c)

Measure of Weight

10 milligrams = 1 centigram
10 centigrams = 1 decigram
10 decigrams = 1 gram
10 grams = 1 decagram
10 decagrams = 1 hectogram
10 hectograms = 1 Kilogram
10 Kilograms = 1 myriagram
100 Kilograms = 1 quintal
10 quintals = 1 metric ton

Also, **1 Kilogram = 1000 grams ; 1 Quintal = 100 Kilograms**

Although there are different units for measurement of length, area, weight, volume etc, but usual units for measurement of length are millimetre (mm), centimetre (cm), metre (m) kilometre (km). Usual units for measurement of area are square metre (Sq.m) and hectare. The units for measurement of Capacity of liquids are normally taken as 'litre', and 'millilitre'. The usual units for measurement of weight are gram, kilogram and quintal.

Exercise—1

1. Express the following in metres :

(a) 5 Km. (b) 2 Km. 10 m (c) 3 decametres 1 m.
(d) 1 hectometre 3 decametre (e) 1 Km. 7 decametres.

2. Express the following in kilometres :

(a) 2000 m (b) 7256 m (c) 19 m (d) 234 decametres.

3. Express the following in grams :

(a) 2 kg. (b) 5 hectograms 3 grams (c) 1 kg. 5 decagrams (d) 5 gm 2 milligrams (e) 3 decigrams 1 milligram (f) 1 quintal.

4. Express the following in litres :

(a) 5 litres 2 decilitres (b) 2 decalitre 5 decilitres
(c) 5 kilolitres 1 litre (d) 5 millilitre (e) 5 decalitres 4 litres

5. Convert the following into kilograms :

(a) 632 gms (b) 1 quintal 50 kg (c) 5 hectograms 3 decagrams (d) 5 metric tons (e) 1 quintal 10 kg 500 grms.

6. Express the following into mixed units :

(a) 625 millimetres (b) 5240 gms (c) 3240 cms
(d) 1220 millilitres (e) 15268 mm (f) 1000 mg.

7. Add the following :

(a) 2 km 5 decametre, 6 hectometre 8 m, 1 km 5 hectemetre, 5 m.
(b) 5 m 6 cm, 8 decimetre, 6 m 4 cm, 2 m 5 cm.
(c) 1 quintal, 75 kg 500 gm, 35 kg 700 gm, 200 gm.

8. Subtract :

(a) 7 kg 70 gm − 1 kg. 80 gm.
(b) 17 m. 80 cm − 5 m. 90 cm.
(c) 3 litre 55 ml − 1 litre 65 ml.

9. Multiply :

(a) 2 km 62 m by 15.
(b) 3 kg 250 gm by 95.
(c) 1 litre 200 millilitre by 120.

10. Divide :

(a) 1 quintal 32 kg by 24.
(b) 45 km by 75.
(c) 427 litres by 35.

11. 1m 55 cm. of cloth is required to make one shirt for Dipu and 2 m 36 cm of cloth is required to make one frock for Soma. How much cloth will be required to make 2 shirts for Dipu and 3 frocks for Soma ?

12. Electrical poles are being installed at a regular interval of 150 m apart in a new road of Bishnupur block. Find the number of poles required for the road 6 km. long.

13. For a picnic party of 50 boys, 6 kg. 250 gm. of rice and 3 kg. 750 gm. of pulse are required. If each boy brings same quantity of rice and same quantity of pulse, how much rice and pulse each boy will have to bring ?

14. 2 kg. 700 gm. of tea at the rate of Rs. 70 per kg. is blended with 3 kg. 300 gm. of tea at the rate of Rs. 100 per kg. What is the price of blended tea per kg?

15. The cooperative store of Mohanpur village has to distribute 10 bags of sugar, each bag containing 1 quintal 4 kg. of sugar. If each person receives 400 gms of sugar, how many persons will receive sugar at one time?

16. 2 milllitres of kerosene oil are leaking out per second through a hole of a tin containing 10 litres of kerosene. How much oil is left in the tin after one hour?

17. 2 millilitres of liquid insecticides are used for 5 sq.m of land. How much insecticide is needed for 1 hectare of land at the same rate?

Exercise—2

1. Write down the numbers between 1 to 20 which are not prime.
2. Write down the prime numbers between 30 to 60.
3. Which of the following are prime numbers?
   37, 39, 43, 51, 67, 71, 77, 123, 127, 129.
4. Find the factors of 16.
5. Find the factors of 75.
6. Find the prime factors of 144.
7. Write down the prime factors of 612.
8. What are the common prime factors of 45 and 60?
9. What are the common prime factors of 325 and 390

10. Resolve into prime factors :

(a) 1250, (b) 1728, (c) 3780, (d) 4950, (e) 10125.

11. By what prime numbers may 192 be divided, so that the remainder will be 5 in each case ?

12. Find the numbers between 60 and 150, by which 715 is divisible ?

13. Determine whether 131 is a prime or a Composite number.

14. 455 mangoes are equally distributed among some boys and girls. Find the possible numbers of boys and girls.

Exercise—3

Find the Highest Common Factors (H. C. F.) by factorisation.

(1) 48, 64 ; (2) 120, 160 ; (3) 84, 98, 112 ; (4) 143, 165, 176 ; (5) 96, 160, 288 ; (6) 170, 204, 510 ; (7) 350, 432, 504 ; (8) 625, 750, 1225 ; (9) 24, 60, 288, 366 ;

(10) Re 1·80 p, Rs. 2·70 p, Rs. 4·20 p.

(11) 2 kg 200 gm. 1 kg 980 gm.

(12) 1m 3 decimetres, 1 m 9 decimetres 5 cm. 2 m 6 decimetres.

(13) 1 litre 820 ml ; 1 litre 40 ml.

Find the Least Common Multiple by factorisation.

(14) 16, 20, 48 ; (15) 22, 33, 25, 45 ; (16) 21, 35, 84, 91 ; (17) 126, 145, 210 ; (18) 55, 132, 165, 220 ; (19) 510, 595, 850 ; (20) 374, 442, 494 ; (21) 570, 665, 798 ;

(22) Rs. 1·65 p., Rs. 2·31 p., Rs. 2·75 p.

(23) 3 m. 25 cm, 2m 21 cm, 2 m 25 cm.

(24) 1 metric ton, 6 quintal, 75 kg.

Exercise—4

1. What do you mean by "$\frac{3}{5}$th of a bamboo pole"? If the bamboo pole is 15m in length, then how much is $\frac{3}{5}$th of the pole? Find the length of the remaining part?

2. The price of $\frac{2}{3}$rd of a metallic rod is Rs 2. What is the cost of the whole rod? What is the cost of $\frac{1}{5}$th of the rod?

3. Rahim had Rs. 4 left with him, after paying $\frac{4}{5}$th of his money for purchasing rice. With how much money did he go to the market?

4. 2 litres of milk were distributed equally among 5 boys and 3 litres of milk were distributed equally among 7 girls. Who has received more milk between a boy and a girl and by how much?

Express the following fractions with lowest common denominator.

(5) $\frac{2}{3}, \frac{3}{5}, \frac{1}{15}$; (6) $\frac{7}{48}, \frac{5}{24}, \frac{2}{3}, \frac{3}{32}$;

(7) $\frac{5}{11}, \frac{7}{12}, \frac{8}{33}$; (8) $\frac{3}{5}, \frac{8}{25}, \frac{9}{35}, \frac{6}{7}$.

Arrange the following in ascending order of magnitude.

(9) $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{6}{7}$; (10) $\frac{2}{5}, \frac{3}{7}, \frac{8}{9}, \frac{3}{10}$;

(11) $\frac{5}{12}, \frac{3}{8}, \frac{4}{7}, \frac{5}{9}$; (12) $\frac{1}{3}, \frac{3}{5}, \frac{2}{7}, \frac{9}{10}$;

Simplify :—

(13) $(\frac{1}{3}+\frac{1}{2})\times\frac{5}{2}\div\frac{1}{2}$; (14) $(\frac{3}{4}+\frac{5}{7})\div 11\frac{5}{7}\times 6\frac{2}{3}$

(15) $(\frac{2}{5}+\frac{3}{15})\div(\frac{1}{20}+\frac{1}{30})+(\frac{2}{5}-\frac{3}{15})\div(\frac{1}{20}-\frac{1}{30})$

Exercise—5

1. Convert into decimal:

(a) $\frac{2}{5}$; (b) $\frac{5}{8}$: (c) $3\frac{3}{5}$; (d) $\frac{41}{5}$; (e) $2\frac{1}{8}$; (f) $4\frac{3}{4}$;

2. **Express as vulgar fraction :**

(a) 0·01 ; (b) 0·125 ; (c) 2·4 ; (d) 0·005 ; (e) 14·85
(f) 2·01 ; (g) 0·00143.

3. Find the sum:

(a) $0{\cdot}301+0{\cdot}197+2{\cdot}075+14{\cdot}79$
(b) $12{\cdot}03+125{\cdot}213+2{\cdot}685+1{\cdot}013$
(c) $3{\cdot}03+112{\cdot}879+13{\cdot}021+2{\cdot}52$
(d) $273{\cdot}815+{\cdot}0231+6{\cdot}0+{\cdot}093$

4. Find the result of subtraction

(a) $125{\cdot}91-0{\cdot}952$; (b) $0{\cdot}8931-0{\cdot}231$
(c) $16{\cdot}45-2{\cdot}032$

5. Find the product

(a) $6{\cdot}7\times10$; (b) ${\cdot}302\times100$; (c) ${\cdot}0312\times1000$
(d) $4{\cdot}832\times14$; (e) $.2717\times20$; (f) ${\cdot}0115\times25$
(g) $2{\cdot}415\times34$

6. Find the quotient

(a) $3.6 \div 10$; (b) $35 \div 100$; (c) $1{\cdot}012 \div 1000$
(d) $48{\cdot}72 \div 12$; (e) $0{\cdot}517 \div 11$; (f) $88{\cdot}83 \div 30$
(g) $2{\cdot}565 \div 19$.

7. Evaluate :

(a) 0·75 of 4 kg. ;

(b) ·066 of Rs. 1200

(c) 1·25 of 5 litres

(d) $\frac{42.3 \times 1.5}{0.47}$

(e) $\frac{{\cdot}06 \times 0{\cdot}3 \times 0{\cdot}14}{70 \times 7{\cdot}2}$

8, Express 5 cm as decimal of 1 m.

9. Express 50 p as decimal of Rs. 5.

10. A group of girl students went on tour by bus. The bus fare of each student was Rs. 2.46 ; and total bus fare was Rs. 123. How many students were there?

11. 0.06 part of a bucket was filled with water. If 2 litters of water was taken out of the bucket, then, 0.35 part of it remains filled with water. Find the total capacity of water in the bucket, when it is completely filled.

12. 0.6 part of students of Raipur Secondary School are boys and the others are girls. One day, 0.2 parts of boys and 0.25 parts of girls were absent. If the number of absentee boys exceeded the number of absentee girls by 10, find the total number of students in the school.

Exercise—6

1. The cost of a dozen of lemons is Rs 4.44 Find the cost of (i) one lemon (ii) 5 lemons (iii) 2 lemons.

2. 9 metres of cloth is required for making 3 garments of same size. How many garments of the same size can be made from 15 metres of cloth of the same kind?

3. A motor car takes 10 mins to travel 11 kms at a uniform speed. Find the distance in Km it travels in (i) 25 minutes (ii) 1 hour.

4. The cost of $2\frac{1}{2}$ quintals of coal is Rs. 450. Find the cost of (i) 40 kg. (ii) 1 quintal and (iii) 1 metric ton of coal.

5. If cost of painting a wall of 1000 sq.metres is Rs. 20.00, how many square metres of wall can be painted at Rs. 225?

6. 20 farmers can harvest a plot in 9 days. In how many days can 15 farmers harvest the same plot?

7. The cost of $\frac{1}{5}$ th of a land is Rs. 925.50 p. How much will be the cost of $\frac{1}{3}$rd of the same land?

8. A students' hostel of 50 students has provisions for 30 days. 25 students left the hostel after 10 days. How long would the food last for the remaining students?

9. 25 labourers earn Rs. 3100 in 4 days. Find the earnings of 12 labourers in 15 days.

10. The price of 15 cows is same as that of 10 buffaloes. The total price of the cows and the buffaloes together is Rs. 51750 What is the price of a cow and also that a buffallo?

11. A bread of weight 100 gms costs 80 paise, if the cost of 1 quintal of flour is Rs. 600 Assuming that the other expenditure

remains same, what would be the cost of 100 gm. of bread if the flour costs Rs. 640 per quintal.

12. A reservoir is filled with water by a pipe. 1000 litres of water rushes through the pipe in 16 mins. Find the total capacity of the reservoir if $\frac{2}{7}$ th part of it is filled in 45 minutes.

CHAPTER 2

Divisibility

If you want to distribute 12 blackberries to three of your friends equally; how many blackberries will each get? Each friend will get 4 blackberries, as 12 can be represented as three groups of four only. Mathematically speaking, we say that 12 is exactly divisible by 3. But if you want to distribute 10 blackberries to three of your friends equally, you will not be able to do that, as 10 is not exactly divisible by 3. Besides 3, 12 is also divisible by 2, 4 and 6. Again, 12 is divisible by 12 and 1. In fact each number is exactly divisible by 1 and the number itself.

Now, is it possible to find out whether a certain number is divisible by another number, without actually performing the division? There are certain tests, by which it is possible to ascertain, whether a number is divisible by other numbers. These are known as divisibility tests.

The criteria of divisibility may be summarized as follows :

(i) A number is divisible by 2, when its unit's place contains zero or an even integer. For example, 192, 326, 1910 etc. are divisible by 2.

(ii) A number is divisible by 3, if the sum of its digits is divisible by 3. The sum of the digits of the number 153 is $(1+5+3)=9$; 9 is divisible by 3. Hence 153 is exactly divisible by 3. Thus we can say the numbers 102, 1752, 2574 also are divisible by 3.

(iii) A number is divisible by 4, if the last two digits of the number are divisible by 4; or last two digits are zero. For example, 324, 1528, 1700 divisible by 4. State whether the numbers which are divisible by 4 are also divisible by 2.

(iv) A number is divisible by 5, if the digit in the unit's place of the number is zero or 5; for example 195, 1720, 2335 etc.

(v) If a number is divisible by 2 and 3. then it is divisible by 6 ; for example 192, 3732, 28770 etc.

(vi) A number is divisible by 8, when its last three digits are divisible by 8 or last three digits are zero ; for example 2720, 3112, 9000 etc.

(vii) A number is divisible by 9, if the sum of its digits is divisible by 9 ; for example 1080, 3357, 9284 etc.

If a number is divisible by 9, will it be divisible by 3 also ?

On the other hand, will a number be divisible by 9 if it is divisible by 3 ?

(viii) A number is divisible by 10, if the digit in the unit's place is zero ; for example, 2320, 1680 etc.

If a number is divisible by 10 then what are the other numbers by which the former number will certainly be divisible ?

(ix) A number is divisible by 11, if the difference between the sum of its digits in the odd places and the sum of its digits in the even place is either zero or is divisible by 11 ; for example 31889. The sum of the digits in the odd place is $3+8+9=20$ and the sum of the digits in the even places is $1+8=9$. The difference is $20-9=11$. Hence the number 31889 is divisible by 11. In 3234, the difference of the sum of the digits in the odd places and sum of the digits in the even places is zero. Hence the number 3234 is divisible by 11.

From the rules stated above, we can also examine the divisibility by some other large numbers. For example, if a number is divisible by both 3 and 4, then the number is divisible by 12 ; if a number is divisible by 2 and 9, the number is divisible by 18.

From the above statements, is it possible to conclude that if a number is divisible by 12, then it is divisible by 6. Or, if a number is divisible by 18, then it is divisible by 3 and 6 ?

It is better to test the divisibility of a number by 7, 13, 17, 19, 23, 29 etc. by exact division.

Exercise—7

1. Determine the divisibility of the following numbers by the numbers as stated.

(a) 173, 183, 546, 940, 6070, 6275 by 3.
(b) 224, 362, 608, 842, 1028, 5400 by 4.
(c) 375, 427, 935, 1600, 4605, 5607, by 5.
(d) 372, 476, 536, 1074, 5274, 7832 by 6.
(e) 119, 235, 343, 2221, 2401, 4607 by 7.
(f) 136, 340, 536, 2064, 2142, 7000 by 8.
(g) 126, 327, 432, 1737, 2184, 8010 by 9.
(h) 242, 321, 341, 1234, 2618, 60907 by 11.
(i) 512, 672, 1620, 3690, 2058, 20736 by 12.
(j) 225, 1080, 1125, 2590, 3645, 4375 by 15.
(k) 702, 1620, 1764, 2673, 3060, 5218 by 18.
(l) Which of the numbers above are divisible by 10 ?

2. Which of the following numbers are divisible by 2, 3, 4, 5, 6, 8. 9. 10 and 11 :

(a) 132, (b) 105, (c) 423, (d) 825, (e) 1617, (f) 2032, (g) 2079, (h) 3210, (i) 5621, (j) 6870.

Factors and Multiples

We know that $2 \times 3 = 6$.

Here 6 is completely divisible by 2 and 3. Hence 2 is a **Factor** of 6, and 3 is also **factor** of 6.

Again, if we multiply 2 by 3 we get 6. Hence 6 is a **multiple** of 2, 6 is a **multiple** of 3.

When we say, 6 is a multiple of 2, we mean that we can divide 6 by 2 and leaves no remainder.

Hence if a number be divided by another number and leaves no remainder, then the second number is known as the factor of first number and the first number is called a multiple of the second number.

For example, 6 is a factor of 18 as 18 is exactly divisible by 6, and 18 is a multiple of 6. Other factors of 18, besides 6, are 1, 2, 3, 9 and 18. But besides 18, there are many other multiples of 6 ; for example, 6, 12, 24, 30, 36...... etc.

The following table shows the factors and the multiples of some numbers.

Numbers	Factors	Multiples
2	1, 2	2, 4, 6, 8, ...
3	1, 3,	3, 6, 9, 12, ...
4	1, 2, 4,	4, 8, 12, 16, ...
5	1, 5,	5, 10, 15, 20, ...
6	1, 2, 3, 6	6, 12, 18, 24, ...
7	1, 7,	7, 14, 21, 28, ...
8	1, 2, 4, 8,	8, 16, 24, 32, ...
9	1, 3, 9,	9, 18, 27, 36, ...
11	1, 11,	11, 22, 3., 44, ...

From the above table, it appears that the factors of some numbers (e.g. 2, 3, 5, 7, 11) are only and the number itself.

The whole number that has only 1 and itself as factors is called a **Prime number.** The whole number that has factors other than 1 and the number itself is called a **Composite number.** Examples are 4, 6, 8, 12, etc.

We observe the special charecteristics of 1. The number 1 is unique. It is a factor of every number. The number 1 is neither prime nor composite.

State whether the number 2 is prime.

Can we say whether the even numbers 4, 6, 8, etc. are prime or not ?

Test for prime number :

We can apply test of divisibility to find whether a number is prime or not. Applying test of divisibility, we can at once find whether a number is divisible by 2, 3, 5 and 11 or not. Now by dividing one after another by 7, 13, 17, 19, 23, 29 it can be seen whether the number is divisible by 7, 13, 17, 19, 23, 29 or not. If the number is not exactly divisible by one of these numbers, the process is continued till the quotient becomes less than the divisor. Then we can say that the number in reference must be prime.

Note that for testing whether a number is prime or not, the divisibility of the number by known prime numbers 2, 3, 5, 7, 11, 13, 17, 19, 23, 29 etc. are considered. It is of no use to see if the number is divisible by some composite number, for if it divisible by a composite number, it must have been divisible by its prime factors. For if it is divisible by 8 or 18, it must have been seen previously that it is divisible by 2 or 3 and thus it would have been ascertained earlier whether it is a prime number or not.

Examples :—

Ex. Find whether 331 is a prime number or not.

By the rules of divisibility, it is seen that the number is not divisible by 2, 3, 5, or 11. So it is to be divided by 7, 13, 17, 19 one after another.

$$
\begin{array}{rl}
7\,) & 331\,(\,47 \\
 & \underline{28} \\
 & 51 \\
 & \underline{49} \\
 & 2
\end{array}
\qquad\qquad
\begin{array}{rl}
13\,) & 331\,(\,25 \\
 & \underline{26} \\
 & 71 \\
 & \underline{65} \\
 & 6
\end{array}
$$

$$
\begin{array}{rl}
17\,) & 331\,(\,19 \\
 & \underline{17} \\
 & 161 \\
 & \underline{153} \\
 & 8
\end{array}
\qquad\qquad
\begin{array}{rl}
19\,) & 331\,(\,17 \\
 & \underline{19} \\
 & 141 \\
 & \underline{133} \\
 & 8
\end{array}
$$

Thus 331 is not divisible by any one of these numbers and it is seen that the quotient when divided by 19 is less than 19. Thus further division by other prime numbers greater than 19 is not necessary. [The reason is that if 331 was divisible by numbers greater than 19, then it would have been divisible by a number less than 19.]

Prime factors :—

The factors of a number which are prime are known as prime factors of that number. For example,

$48=2\times2\times2\times2\times3$.

Therefore, prime factors of 48 are 2. 2, 2, 2 and 3. It can be said that the independent prime factors of 48 are 2 and 3.

It is worth mentioning here that the repeated multiplication of the type $2\times2\times2\times2$ can be written as 2^4 This is read as '2 to the power of 4'. Similarly. $2\times2\times2=2^3$ (2 to the power of 3). $3\times3\times3\times3\times3=3^5$ (3 to the power of 5).

Hence $48=2\times2\times2\times2\times3$

$=2^4\times3$.

$100=2\times2\times\ \times5\times5=2^2\times5^2$

$125=5\times5\times5=5^3$

$144=2\times2\times2\times2\times3\times3=2^4\times3^2$

To find the prime factors of number :—

To find the prime factors of any given number a successive division test is applied. First we divide the given number by one of its prime factors; then we divide the quotient by one of its prime factors and this process is continued until a prime number is left as the last quotient. Then all the divisors and the last quotient form a set of prime factors of the given number.

Example—

Ex. Find the prime factors of 756.

$$\begin{array}{r|l} 2 & 756 \\ \hline 2 & 378 \\ \hline 3 & 189 \\ \hline 3 & 63 \\ \hline 3 & 21 \\ \hline & 7 \end{array}$$

∴ The prime factors of 756 are 2, 2, 3, 3, 3, 7

We can write,

$756=2\times2\times3\times3\times3\times7$

$=2^2\times3^2\times7$

Ex. Express 2475 as the product of prime factors.

$$\begin{array}{r|r} 3 & 2475 \\ \hline 3 & 825 \\ \hline 5 & 275 \\ \hline 5 & 55 \\ \hline & 11 \end{array}$$

Hence,

$2475=3\times3\times5\times5\times11$

$=3^2\times5^2\times11$

Numbers prime to each other :—

If two numbers have no common prime factors, then the numbers are called prime to each other (or relatively prime). For example, the prime factors of 15 are 3, 5 and that of 16 is 2. Therefore, 15 and 16 have no common prime factor. So 15 and 16 are prime to each other. Observe that none of the numbers 15 and 16 are prime. Two prime numbers will always be relatively prime.

Exercise—8

1. (a) Write down the prime numbers between 10 and 50
   (b) Find the prime numbers between 100 and 120.

2. Find the prime factors of.

(a) 108, (b) 252 (c) 273 (d) 343 (e) 504
(f) 1326 (g) 3300 (h) 5940 (i) 19800.

3. Express the following as products of prime factors.

(a) 132 (b) 225 (c) 702 (d) 1050 (e) 3150
(f) 740 (g) 21840.

4. Determine whether the following numbers are prime or not :

(a) 271 (b) 557 (c) 923 (d) 1411.

5. Are the following pairs of numbers prime to each other ?

(a) 42, 55 (b) 112, 153 (c) 144, 245 (d) 175, 462
(e) 220, 819.

6. Find the product of the following :—

(a) $2^3 \times 3$ (b) $2 \times 3^2 \times 5^2$ (c) $2^3 \times 3^2 \times 7$
(d) $2^4 \times 3^3 \times 5^2$ (e) $2^3 \times 3^4 \times 5^3$.

Highest common factor :—

If two numbers have same factor, it is termed as a common factor of these two numbers. The factors of 18 are 1, 2, 3, 6, 9 and 18. The factors of 30 are 1, 2, 3, 5, 6, 10, 15 and 30. The factors which are common to 18 and 30 are 1, 2, 3, and 6. The highest factor among these factors is 6. Thus the highest common factor of 18 and 30 is 6.

Again, $18 = 2 \times 3 \times 3$
$30 = 2 \times 3 \times 5$

We see that the prime factors of 18 are 2, 3, and 3 and prime factors of 30 are 2, 3, and 5. The common factors are 2, 3 and 6. Observe that $6 = 3 \times 2$.

Thus we see that highest common factor is the product of the common prime factors of the given numbers.

If there is only one prime factor, which is common to both the numbers, then that is the highest common factor of the two numbers.

In abbreviation, we write highest common factor as H. C. F.

We can find the H. C. F. of two numbers in a short method by factorising simultaneously.

Example 1—Find the H.C.F. of 24, 36.

$$\begin{array}{r|l} 2 & 24,\ 36 \\ \hline 2 & 12,\ 18 \\ \hline 3 & 6,\ 9 \\ \hline & 2,\ 3 \end{array}$$

$\therefore$ Required H. C. F. $= 2 \times 2 \times 3 = 12$

For finding H. C. F. of more than two numbers one has to resolve the numbers into its prime factors. The product of all the common factors is the required H .C. F.

Example 2—Find the H.C.F. of 72, 60, 96.

$$\begin{array}{r|l} 2 & 72 \\ \hline 2 & 36 \\ \hline 2 & 18 \\ \hline 3 & 9 \\ \hline & 3 \end{array} \qquad \begin{array}{r|l} 2 & 60 \\ \hline 2 & 30 \\ \hline 3 & 15 \\ \hline & 5 \end{array} \qquad \begin{array}{r|l} 2 & 96 \\ \hline 2 & 48 \\ \hline 2 & 24 \\ \hline 2 & 12 \\ \hline 2 & 6 \\ \hline & 3 \end{array}$$

$$\begin{aligned} \therefore\ 72 &= 2 \times 2 \times 2 \times 3 \times 3 \\ 60 &= 2 \times 2 \times 3 \times 5 \\ 96 &= 2 \times 2 \times 2 \times 2 \times 2 \times 3 \end{aligned}$$

Prime factors common to 72, 60, and 96 are 2, 2 and 3, Hence H. C. F. $= 2 \times 2 \times 3 = 12$.

In case of large numbers, which can not be easily resolved into prime factors, method of division is applied for finding H. C. F. Divide the greater number by the smaller number. Take the remainder as divisor and the first divisor as dividend, and perform the operation of division. This process is repeated until there is no remainder. The last divisor is the H. C. F. of the two given numbers. For example, let us find the H. C. F. of 18 and 30 by the method of division.

$$\begin{array}{rrrl}
18\,) & 30\,(& 1 & \\
 & 18 & & \\
\hline
 & 12\,) & 18\,(& 1 \\
 & & 12 & \\
\cline{2-4}
 & & 6\,)\,12\,(& 2 \\
 & & 12 & \\
\cline{3-4}
 & & \times &
\end{array}$$

Required H. C. F. of 18 and 30 is 6.

Observe that the H. C. F. of 18 and 30 has been found 6 by the method of factorisation previously.

If the H. C. F. of three numbers is to be found out, first find out the H. C. F. of any two numbers. Then find out the H. C. F. of the above H. C. F. and the third number. The last H. C. F. will be the required H. C. F. of the three given numbers. If there are more than three numbers. H. C. F. can be found out by the same process.

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Fxamples.

Ex—3. Find the H. C. F. of 125 and 475:

```
125 ) 475 ( 3
      375
      ---------
      100 ) 125 ( 1
            100
            ---------
             25 ) 100 ( 4
                  100
                  ------
                   ×
```

Required H. C. F. is 25

Ex—4. Find the H. C. F. of 144, 240 and 612.

```
144 ) 240 ( 1
      144
   -----------
       96 ) 144 ( 1
             96
        ---------
             48 ) 96 ( 2
                  96
                ------
                   ×
```

```
48 ) 612 ( 12
     48
   --------
     132
      96
   ---------
      36 ) 48 ( 1
           36
         --------
           12 ) 36 ( 3
                36
              ------
                 ×
```

Hence required H. C. F. = 12

Remark : It is convenient to find the H. C. F. of two smaller number first.

Ex—5. Find the H. C. F. of 324, 480, 618, 710.

```
324 ) 480 ( 1                          12 ) 618 ( 51
      324                                   60
      ---------                             ------
      156 ) 324 ( 2                          18
            312                              12
            ---------                       ------
             12 ) 156 ( 13                 6 ) 12 ( 2
                  12                             12
                  -----                          ----
                   36                             ×
                   36
                  -----
                    ×
```

```
 6 ) 710 ( 118
     6
     -----
     11
      6
     -----
     50
     48
     -----
     2 ) 6 ( 3
         6
        ---
         ×
```

Required H.C.F. = 2

H.C.F. of compound quantities :

To find the H.C.F. of Similar compound quantities, all the compound quantities are to be expressed in the same unit and then the H.C.F of the numbers connected with that unit is found. This, when expressed in the above unit gives the required H.C,F.

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Generally, the H.C.F. is found by converting them in the smallest unit.

Example 6—Find the H.C.F. of 3 kg. and 400 gm.

3 kg = 3 × 1000 gm = 3000 gm.

We will find the H.C.F. of 3000 gm and 400 gm.

400) 3000 (7
2800
200) 400 (2
400

H.C.F. of 3000 and 400 is 200

Hence H.C.F. of 3 kg. and 400 gm = 200 gm.

Note. (1) The H.C.F. of two prime numbers or relatively prime number is always 1. For example H.C.F. of 11 and 17 is 1, or that of 16, 27 is 1.

(2) If one of the numbers is the factor of all the numbers, then that number is H.C.F. of all the numbers.

Exercise—9

1. Find H.C.F. by the method of factorization.

(a) 10, 15 (b) 20, 30 (c) 30, 45 (d) 40, 60 (e) 20, 50
(f) 5, 25, 75 (g) 20, 30, 60 (h) 40, 60, 100
(i) 45, 60, 75 (j) 22, 66, 88 (k) 120, 140, 160
(l) 360, 400 (m) 196, 280 (n) 78, 182, 195
(o) 84, 140, 308 (p) 102, 595, 357.

2. Find H. C. F. by the method of division.

(a) 296, 703 (b) 867, 5967, (c) 300, 3996 (d) 160, 165, 305 (e) 306, 405, 450, 1107 (f) 9317, 5445 (g) 399, 665, 1463 (h) 905, 1510, 1057 (i) 627, 779, 1843, 3762 (j) 427; 1159, 1281, 3416 (k) 8346, 27948 (l) 996, 1245, 38263.

3. Find the H. C. F. of the following :—

(i) Re. 1·20 P., Rs. 3·40 P., Rs. 4·80 p.

(ii) Rs. 5·95 P.. Rs. 7·91. P., Rs. 10·43 P.

(iii 6 Kilogram 6 hectogram 5 decagram, 1 kilogram 1 hectogram 9 decagram 7 gram.

(iv) 9 kg. 320 gm., 1 kg. 398 gm.

(v) 6 m.: 25 cm., 9 m 50 cm., 12 m. 52 cm.

(vi) 3 m. 96 cm,. 6 m. 9 dm. 3 cm.

(vii) 12 quintal, 9 quintal 10 kg., 50 kg.

(viii) 1 litre 603 ml., 3 litre 664 ml.

(ix) 5 decilitre 9 centilitre 5 ml., 7 decilitre 9 centilitre 1 ml.

4. Two tankers contain 868 litres and 980 litres of diesel oil. Find the maximum volume of a container that can be used for measuring the oil in the two tankers in complete number of times.

5. Find the greatest number that can divide 306, 810 and 2214 leaving no remainder in each case.

6. The length and breadth of a hall is 19m 20cms and 14m respectively. What would be the maximum length of a square tile, if the floor of the hall is to be covered entirely by such tiles ?

Least Common Multiple

We know that a multiple of any number is exactly divisible by that number. For example multiples of 3 are 3, 6, 9, 12, 15, 18,.......and so an. Each multiple of 3 is completely divisible by 3 and we find that a number has an infinite number of multiples. The multiple of 4 are 4, 8, 12, 16, 20, 24, 28, 32, 36. Similarly multiples of 6 are 6, 12, 18, 24, 30, 36 etc. Among the multiples of 4 and 6, we find some common multiples e. g. 12, 24, 36,.......etc. The number of such common multiples is infinite. Of all the common multiples 12 is the least and is termed as least common multiple or in short L. C. M. of 4 and 6.

Hence the least common multiple (L. C. M.) of two or more number is the smallest number which is a multiple of each of the two numbers.

Note :— H. C. F. is highest common factor.
L. C. M. is lowest common multiple.

To find the L. C. M- of two or more numbers by prime factorization.

Since a multiple of a number is divisible by that number, the multiple must be divisible by the prime factors of the number. Hence to find the L. C. M. of two or more numbers, we first express each of the numbers as the product of its prime factors. The L. C. M. is then determined by the continued product of the prime factors of each of the numbers, each prime factor is to be taken maximum times it appears in any of the number. If a prime factor is taken more than the maximum number of times of its appearance in any of the number, the result will be a common multiple instead of least common multiple. On the other hand if a prime factor is taken less than the maximum number of times of its appearance in one of the numbers, the product will not be at all a common multiple. The following example will help to understand the process.

Example : Find the L. C. M. of 16, 24, 48

$$\begin{aligned}16&=2\times2\times2\times2=2^4\\24&=2\times2\times2\times3=2^3\times3\\48&=2\times2\times2\times2\times3=2^4\times3.\end{aligned}$$

The prime factor 2 has appeared 4 times in 1st number, 3 times in 2nd number and 4 times in the third number. For finding L. C. M. 2 will have to be taken 4 times. If instead 2 is taken 5 times then the continued product would have been $2^5\times3=32\times3=96$. Although 96 is a common multiple. of 16, 24, 48, but it is not the least common multiple. If 2 is taken 3 times instead of 4 times, the continued product is $2^3\times3=24$, which is not at all a common multiple as it is not divisible by 48,

Hence L. C. M. $=2^4\times3=16\times3=48$.

Example 2. Find the L. C. M. of 45, 75, 125.

$$\begin{aligned}45&=3\times3\times5=3^2\times5\\75&=3\times5\times5=3\times5^2\\125&=5\times5\times5=5^3.\end{aligned}$$

Required L. C. M. $=3^2\times5^3=9\times125=1125$.

Shorter process of finding L. C. M.

Arrange the numbers whose L. C. M. is to be found out, in a line side by side, separating the numbers by using coma (,). Divide the number by a prime factor which will divide at least any two of the given numbers. The quotients and the numbers which are not divisible by the above factors are written in the next line each quotient being below the dividend and other numbers are to be written below the corresponding numbers. Similar operation of dividing the numbers by prime factors common to two or more numbers is performed until the set of quotient left are relatively prime to each other. Then all the factors and all the numbers in the last row (which are relatively prime) are multiplied together to get the L. C. M. of the numbers.

Example 3. Find the L.C.M. of 27, 35, 45 and 49.

$$\begin{array}{r|llll}
3 & 27, & 35, & 45, & 49 \\ \hline
3 & 9, & 35, & 15, & 49 \\ \hline
5 & 3, & 35. & 5, & 49 \\ \hline
7 & 3, & 7, & 1, & 49 \\ \hline
 & 3, & 1, & 1, & 7
\end{array}$$

Required L. C. M.$=3\times3\times5\times7\times3\times7=6615$.

Note : (1) The L. C. M. of a set of prime numbers or relatively prime numbers is their product only e. g. the L. C. M. of 2, 5, 7 is $2\times5\times7=70$; the L. C. M. of 4 and 9 is $4\times9=36$.

(2) If one of the numbers is the multiple of all other numbers, then that number is the L.C.M. of all the numbers.

L. C. M. of compound quantities :

In finding the L. C. M. of a set of similar compound quantities, we have to convert each of the quantity to their lowest unit, (expressed in same unit) and then find the L.C.M. of the numbers obtained after conversion. L. C. M. thus obtained is expressed in that unit and can be converted to other units if necessary.

Example : Find the L. C. M. of 5 decimetre, 4 decimetre 5 centimetre.

5 decimetre = 50 cm.
4 decimetre 5 centimetre = 45 cm.

Now L. C. M. of 50, 45 = 450

Required L. C. M. = 450 cm = 45 decimetre.

Relation between L. C. M. and H. C. F. :

Let us take any two numbers 30 and 50.

L. C. M. of 30 and 50 = 150

$$\begin{array}{r|l} 2 & 30,\ 50 \\ \hline 5 & 15,\ 25 \\ \hline & 3,\ 5 \end{array}$$

again $30 = 2 \times 5 \times 3$

$50 = 2 \times 5 \times 5$

Thus H. C. F. of 30 and $50 = 2 \times 5 = 10$

and L. C. M. of 30 and $50 = 2 \times 5 \times 3 \times 5 = 150$.

Note that

L. C. M. of the two number × H. C. F. of the two numbers

$= (2 \times 5 \times 3 \times 5) \times (2 \times 5)$

$= (2 \times 5 \times 3) \times (2 \times 5 \times 5)$

$= 30 \times 50 =$ product of the two numbers

The above result is not only true for the above two numbers, but for any two numbers in general.

For in the above example, the common factors 2 and 5 of the numbers appear both in H. C. F. and L. C. M. and each of the other factors 3 and 5 are present only once in L. C. M.

Then if in the product of L. C. M. and H. C. F. these additional factors are properly adjusted we get the product of the given numbers.

In the above case,

H. C. F. × L. C. M. $= (2 \times 5) \times (2 \times 5 \times 3 \times 5)$

$= (2 \times 5 \times 3) \times (2 \times 5 \times 5)$

= First number × Second number.

Thus we see that for any two numbers H. C. F. × L. C. M. = product of two numbers. Now, is it possible to apply this rule for three numbers also ? In other word, is it true that the

product of L. C. M. and H. C. F. of three numbers is equal of the product of the three numbers. You may try by taking three numbers of any choice and by finding the product of the L. C. M. and H. C. F. and also that of the numbers.

However, in all cases, this rule is very important for two numbers and is sometimes used for finding L. C. M. of two numbers. L. C. M. of two numbers is obtained by dividing the product of the numbers by their H. C. F.

$$\text{Thus L.C.M. of two numbers} = \frac{\text{Product of the two numbers}}{\text{H.C.F. of the two given numbers}}$$

If there are more than two numbers, one has to find L.C.M. of any two numbers. The L.C.M. thus obtained is taken as one number and similar process is applied with L.C.M. of the first two numbers and the next number.

Example. Find L.C.M. of 75 and 125 using their H.C.F.

$75 = 5 \times 5 \times 3$

$125 = 5 \times 5 \times 5$

H.C.F. of 75 and $125 = 5 \times 5 = 25$

Now, the product of the numbers $= 75 \times 125 = 9375$

L.C.M. $= 9375 \div 25 = 375.$

Example 6. Find the L.C.M. of 12, 15 and 18 with the help of their H.C.F.

H.C.F. of 12 and $15 = 3$

Hence L.C.M. of 12 and $15 = 12 \times 15 \div 3 = 60$

Now H.C.F. of 60 and 18 = 6
L.C.M. of 60 and 18 = 60 × 18 ÷ 6 = 180

∴ L.C.M. of the numbers = 180.

Exercise—10

Find the L.C.M. of the following. (oral)

(1) 12, 16 (2) 24, 36 (3) 10, 15, 30, (4) 8, 12, 24 (5) 12, 15, 20, 30.

Find the L.C.M. of the following by factorization.

(6) 20, 25 (7) 36, 48 (8) 63, 84 (9) 96, 120 (10) 147, 252 (11) 35, 49, 91 (12) 30, 36, 40 (13) 42, 48, 56 (14) 165, 264, 286 (15) 676, 702, 936.

Find the L.C.M. of the following by shorter process.

(16) 15, 25, 30 (17) 14, 21, 56 (18) 40, 50, 60
(19) 28, 35, 56, 84 (20) 16, 24, 30, 40, 48
(21) 24, 36, 54, 72. 96.

Find the L.C.M. of the follwing by using H.C.F.

(22) 40, 60 (23) 36, 48 (24) 72, 120 (25) 48, 60, 90 (26) 6, 15, 20, 25 (27) 16, 36, 54, 108.

Find the L.C.M. of the following.

(28) Rs. 2, Rs. 2.40, Rs. 3.20.
(29) Re. 1.26, Re 1.68, Rs. 2.94.
(30) 3kg. 6 hg. 4 dg., 19kg. 6 hg.,
7hg. 2 dg. 8 gm.
(31) 2m. 28 cm., 3 m. 24 cm., 4m. 86 cm.
(32) 6 hrs. 4 mins. 30 secs., 2 hrs. 42 mins.

Find the H.C.F. and L.C.M. of the following numbers and compare their products with product of the numbers.

(33) 90 144 (34) 112, 140 (35) 25, 30, 45
(36) 20, 25, 30, 40.

Miscellaneous problems on L.C.M. and H.C.F. and their solutions.

Example 1. Find the greatest integer which when divides 202 and 321, leaves remainders 4 and 6 respectively.

Since 4 and 6 are the remainders when 202 and 321 are divided by a number, then the number must divide $202-4=198$ and $321-6=315$ exactly.

Hence the required number is the H.C.F. of the 198 and 315.

```
Now   198 ) 315 ( 1
            198
          -----------
          117 ) 198 ( 1
                117
              -----------
               81 ) 117 ( 1
                     81
                   -----------
                    36 ) 81 ( 2
                         72
                       ---------
                         9 ) 36 ( 4
                             36
                           ------
                              ×
```

Hence the required number is 9,

Note : Check your answer by dividing 202 and 321 by 9, whether you get the remainders as 4 and 6 respectively or not.

Example—2. Find the least number of 4 digits which is divisible by 12, 15, 20, 35.

The number divisible by 12, 15, 20 and 35 is also divisible by the L.C.M. of these numbers.

$$\begin{array}{r|l} 2 & 12,\ 15,\ 20,\ 35 \\ \hline 2 & 6,\ 15.\ 10,\ 35 \\ \hline 3 & 3.\ 15,\ 5,\ 35 \\ \hline 5 & 1,\ 5,\ 5,\ 35 \\ \hline & 1,\ 1,\ 1,\ 7 \end{array}$$

L. C. M. $= 2 \times 2 \times 3 \times 5 \times 7 = 420$

Now the least number of 4 digits is 1000.

$$\begin{array}{r} 420\ )\ 1000\ (\ 2 \\ 840 \\ \hline 160 \end{array}$$

If we divide 1000 by 420, the remainder is 160. If the dividend was 420—160 = 260 more then it would have been divisible by 420.

Thus the required number $= 1000 + 260 = 1260$

Note : $1000 - 160 = 840$ is also divisible by 420, but it is not a four digit number.

Example 3. Find the greatest number of five digits which when divided by 16, 24, 30 and 36 leaves a remainder of 10 in each case.

The L. C. M. of 16, 24, 30, 36 = 720 (you work out yourself) ;

The greatest number of five digits = 99999

```
720 ) 99999 ( 138
      720
     ----
      2799
      2160              Now, 99999 - 639
     -----                   = 99360
       6399
       5760
      -----
        639
```

Now the greatest number of five digits divisible by 720 is 99360. As there will be a remainder of 10 in each case, the required number is $99360 + 10 = 99370$.

Example 4. Find the smallest number, from which if I is subtracted, the remainder is divisible by 15, 18, 20, 24.

The L. C. M. of 15, 18, 20, 24 = 360. (you work out yourself);

Required number is $360 + 1 = 361$.

Example 5. L. C. M. and H. C. F. of two numbers are 240 and 16 respectively. If one number is 48, find the other.

$$\text{Required number} = \frac{\text{L. C. M.} \times \text{H. C. F.}}{48}$$

$$= 240 \times 16 \div 48 = 16 \times 240 \div 48$$
$$= 16 \times 5 = 80$$

Example 6. Find the numbers of pieces of greatest length of equal measure that can be cut out from two wires of Aluminium and Copper whose lengths are 9 m 60 cm and 5 m 12 em respectively.

9 m. 60 cm.=960 cm.

5 m. 12 cm.=512 cm.

$$\begin{array}{r|l} 2 & 512,\ \ 960 \\ \hline 2 & 256,\ \ 4\ 0 \\ \hline 2 & 128,\ \ 240 \\ \hline 2 & 64,\ \ 120 \\ \hline 2 & 32,\ \ 60 \\ \hline 2 & 16,\ \ 30 \\ \hline & 8,\ \ 15 \end{array}$$

H. C. F. of 960 and 512 $=2\times2\times2\times2\times2\times2=64$

∴ Greatest length of each piece=64 cm.

Again, the length of the Aluminium wire=960 cm. $=64\times15$ cm.

∴ Number of pieces of Aluminium wire=15

The length of the copper wire=512 cm. $=64\times8$ cm.

Number of pieces of copper wire=8

Exercise—11

1. Find the least number which is divisible by 12, 15, 20 and 25.

2. Find the greatest number that divides 1375 and 4935 leaving no remainders.

3. Find the greatest number that will divide 564 and 630 leaving a remainder 3 in each case.

4. Find the greatest and least number which being subtracted from 5834, will make the result exactly divisible by 20, 28, 32 and 35.

5- What is the greatest number that leaves no remainder while dividing by 984 and 1320 ?

6. Sukla bought some exercise books for Rs. 2.40 .. Her. elder brother bought some exercise books of same type for

Rs. 3.60. What will be the maximum possible price for such an exercise book ? Find the total number of exercise books ; also the number of books bought by Sukla and by her elder brother.

7. Find the least number, when 5 is added to it, the sum is exactly divisible by 16, 24 and 32.

8. Four bells ring at an interval of 45 mins, 1 hr, 1 hr 15 mins and 1 hr 30 mins respectively. All the bells ring simultaneously at 12 noon. When will again they ring simultaneously ? How many times will each bell ring during that interval ?

9. The lengths of two sheets, one of Iron and the other of Copper are 6m. 72 cm. and 9m. 60 cm. respectively. What should be the greatest length of each sheet that can be cut out from the two sheets ? Find the number of pieces of each sheet.

10. A store in a relief camp has 1080 pieces of dhotis, 1620 pieces of sarees and 2700 pieces of shirts. Find the maximum number of family to whom above things can be distributed equally and also find the quantity of dhoties, sarees and shirts each will get.

11. L. C. M. and H. C. F. of two numbers are 1760 and 32 respectively. One number is 160. Find the other number.

12. Find the greatest number of five digits which when added to 100, the sum becomes exactly divisible by 24 and 28.

13. Find the numbers between 300 and 500 which are divisible by 6, 8. 10 and 12.

14. What is the least number that leaves remainders 15, 20, 25, 31 and 43 respectively when divided by 20, 25, 30, 36 and 48 ?

15. Find the greatest number that will divide 2300 and 3500 leaving the remainders 32 and 56 respectively.

16. Find the greatest number that will divide 650, 775 and 1250 leaving the same remainder in each case.

17. Two containers of same size contain 5 kilolitres 432 litres of mustard oil and 5 kilolitres 917 litres of cocoanut oil respectively. Some tins, each of same size are completely filled up by the two types of oil separately. What should be the maximum volume of such a tin and what is the least number of such tins necessary for each kind of oil?

18. The sum of two numbers is 384 and their H.C.F. is 48. What are the probable pairs of numbers?

19. Buses run from Ballygunge Bus depot to Paikpara, Bagbazar, Howrah and Barrackpur respectively at an interval of 10 mins, 15 mins, 20 mins and 25 mins respectively. Four buses, one on each of the four routes started together at 6 A.M. Find the time when the buses in the four routes will start together again. Find the number of buses on each route that leave the depot by that time.

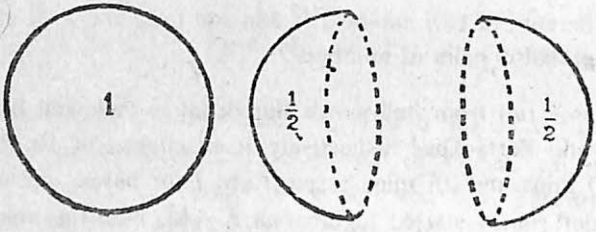
20. What is the least number when 4000 is subtracted from it, the remainder becomes divisible by 7, 11 and 13?

21. What least number is to be added with 5000 to make the sum divisible by 10, 12, 16 and 18?

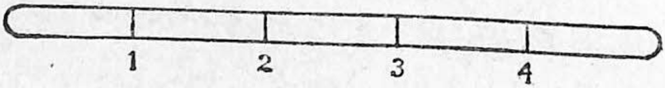
CHAPTER 3

Fractions

Fractions : Sometimes it is necessary to divide a whole thing for proper distribution. Suppose you have only one guava and you give half of it to your brother. How will you do that ? You divide the guava into two equal parts and give a part to your brother. Thus your brother gets one of the equal halves of the guava and this part is known as a fractional part or simply fraction.



The whole guava which is divided into equal parts is known as unity and is symbolized as 1. We have divided the 'whole' guava into two equal parts and each part is written as $\frac{1}{2}$. A fraction is thus usually expressed by two numbers, one over the other divided by a horizontal line segment. The lower number indicates the number of equal parts we are dividing the whole and the upper number iudicates the number of parts we are taking. For example, if a stick is divided into 5 equal parts, then each part is one fifth of the whole number and written as $\frac{1}{5}$.



Similarly 2 parts taken out of 5 equal parts forms $\frac{2}{5}$ (two upon five) of the whole, 3 parts taken out of 5 equal parts forms $\frac{3}{5}$, 4 parts taken out of 5 equal parts forms $\frac{4}{5}$ and finally parts taken together forms the whole stick is $\frac{5}{5}$, which implies the entire stick, which is equal to 1. The number above the line is known as **numerator** and the number below the line is known as **denominator.** In the fraction $\frac{6}{7}$, 7 is the denominator and 6 is the numerator. Thus we see that one or more parts taken from equal division of a thing is expressed by fraction.

Now we know that in the operation of division, the divisor is always taken less than the dividend. Here in discussing fraction, we find that dividend is less than divisor. Thus in previous example 1 guava has been divided by 2 and the quotient is $\frac{1}{2}$ guava or in the case of stick, 1 has been divided by 5 and the quotient is $\frac{1}{5}$. The other interesting point is that there is no use of remainder, when we are considering division as a fraction. Hence fraction can be treated as a division and a similar-symbol can be used e.g. $\frac{1}{2}=1\div2$, $\frac{1}{5}=1\div5$, $\frac{3}{5}=3\div5$, $\frac{6}{7}=6\div7$.

Hence, we can always write, fraction = numerator ÷ denominator.

Example 1. How much paise is $\frac{2}{5}$ th of one Rupee.?

1 Rupee = 100 paise
100 paise ÷ 5 = 20 paise
20 paise × 2 = 40 paise
$\therefore$ $\frac{2}{5}$ th of a rupee = 40.paise.

Example 2. How much is $\frac{3}{10}$ of a metre ?

1 metre = 100 cm
100 cm ÷ 10 = 10 cm
10 cm × 3 = 30 cm
$\therefore$ $\frac{3}{10}$ of a metre = 30·cm.

Proper, Improper and mixed fractions

In our previous discussion we have found that a fraction is nothing but another form of the operation of division. In such a form, the numerator is dividend and denominator is divisor. Thus we have been able to avoid the restrictions in our previous discussion on division that divisor can not be greater than dividend. We can consider that the operation of division, where any one of the denominator and the numerator may be greater or less than the other, is free of the above restriction and in that sense, the process has been a bit generalised. In particular case, when numerator is equal to the denominator then the fraction becomes unity.

In the previous example of the stick, we have seen that $\frac{5}{5}=1$ and in case of guava, $\frac{2}{2}=1$.

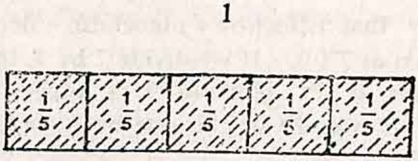
Fractions can be classified into two groups depending on the numerator is less than the denominator, or numerator is greater than or equal to the denominator.

In the fractions $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{7}$, we find that numerator is less than denominator. These fractions in which numerator is less than denominator are termed as **proper fractions.**

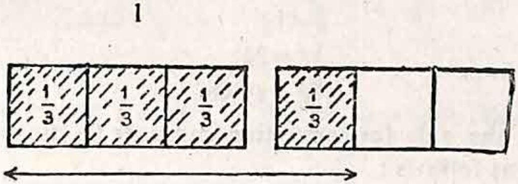
Again, in the fractions $\frac{4}{3}$, $\frac{5}{2}$, $\frac{8}{3}$ numerator is greater than the denominator. Again we can write 1 in the form of fraction by $\frac{2}{2}$, $\frac{3}{3}$. These fractions in which numerator is greater than or equal to the denominator are termed as **improper fractions.**

Let us now consider, the actual implications of these 'improper fractions'. Consider the improper fraction $\frac{4}{3}$ Since the denominator is '3', we are dividing unity into 3 equal parts of which each equal part is $\frac{1}{3}$. We are taking 4 such parts to make $\frac{4}{3}$. Now 3 parts of $\frac{1}{3}$ makes a unity and we still have $\frac{1}{3}$ more. Hence $\frac{4}{3}=1+\frac{1}{3}$. which is written in short as $1\frac{1}{3}$ and is read as 'one and one third'. Similarly if we consider the improper fraction $\frac{5}{2}$, we are considering 5 equal parts of $\frac{1}{2}$. Four equal parts of $\frac{1}{2}$, makes 2 whole or twice of unity and we are still left with another $\frac{1}{2}$. Hence $\frac{5}{2} = 2 + \frac{1}{2}$, or in words '2 and half'.

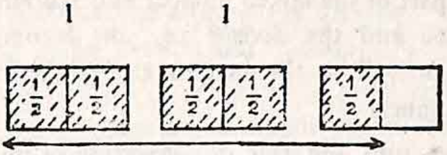
Following diagrams will be helpful for clear understanding of improper fraction.



(Representation of $\frac{5}{5}$)



$\frac{4}{3}$ (Representation of $\frac{4}{3}$)



$\frac{5}{2}$ (Representation of $\frac{5}{2}$)

Numerator $<$ Denominator implies the fraction as proper whose value <1

Numerator $=$ Denominator implies improper fraction whose value $=1$

Numerator $>$ Denominator implies improper fraction whose value >1.

Here <1, means less than 1

$=1$, means equal to 1

and >1, means greater than 1.

A fraction composed of a whole number and a proper fraction is known as mixed number. Examples of some mixed numbers are $1\frac{1}{5}$, $2\frac{1}{3}$, $3\frac{1}{5}$, etc. A mixed number is always greater than 1.

Now, we know that a fraction = numerator ÷ denominator. Thus $\frac{7}{3}$ can be written as $7 \div 3$. If we divide 7 by 3, the quotient is 2 and remainder is 1. Thus in 7, 3 appears twice and remainder is one. If we divide this one by 3, we get the fraction $\frac{1}{3}$. Hence $\frac{7}{3} = 2\frac{1}{3}$.

Similarly, we can write,

$$\frac{8}{5} = 1\frac{3}{5}$$
$$\frac{16}{7} = 2\frac{2}{7}$$
$$\frac{25}{3} = 8\frac{1}{3} \text{ etc.}$$

Hence the rule for cenverting improper fractions into mixed number is as follows :

Divide numerator by denominator of which the quotient will be the integral part of the mixed number and the remainder will be the numerator and the divisor i.e. the denominator of the improper fraction will be the denominator of the fractional part of the mixed number.

Now let us find the rule of conversion of mixed numbers into improper fraction.

We know that in the operation of division, dividend = divisor × quotient + remainder.

Now $3\frac{4}{7}$ is a mixed number. When this is converted to an improper fraction, the improper fraction will have a denominator 7 and have a numerator which when divided by 7 gives a quotient 3 with remainder 4. Thus the numerator $= 7 \times 3 + 4 = 25$.

$$\therefore 3\frac{4}{7} = \frac{25}{7}.$$

Thus the rule of conversion of mixed numbers into improper fraction is as follows. The denominator of the fractional part of the mixed number will be the denominator of the improper fraction and the numerator of improper fraction is obtained by adding the numerator of the fractional part of the mixed number to the product of the integral part and denominator of the fractional part of the mixed number.

Example 1. Express $\frac{123}{5}$ as mixed number.

$$\begin{array}{l} 5\ )\ 123\ (\ 24 \\ \qquad \underline{10\quad} \\ \qquad\ 23 \\ \qquad\ \underline{20\quad} \\ \qquad\ \ \ 3 \end{array}$$

$\therefore$ Hence $\frac{123}{5} = 24\frac{3}{5}$

Example 2. Convert $112\frac{3}{5}$ into improper fraction.

$$112\frac{3}{5} = \frac{5 \times 112 + 3}{5} = \frac{563}{5}$$

We know, fraction = Numerator ÷ Denominator.

If we think fraction as a process of division, we will be able to know some more facts about fraction.

One of the rules of division is that quotient remains unchanged if the divisor and dividend are multiplied or divided by the same number. For example, we know that

$$12 \div 4 = 3.$$

Now $12 \times 6 = 72$ and $4 \times 6 = 24$.
If we divide 72 by 24 we get $\frac{72}{24} = 3$.
Again $96 \div 8 = 12$.
Now $96 \div 4 = 24$ and $8 \div 4 = 2$.
If we divide 24 by 2, we get $\frac{24}{2} = 12$.

You prepare yourselves some more examples and be sure of these rules.

Now by applying these rules we will get some new properties of fractions.

(1) A given fraction $= \dfrac{\text{Numerator}}{\text{Denominator}}$

$= \text{Numerator} \div \text{Denominator}$

$= (\text{Numerator} \times \text{any number}) \div (\text{Denominator} \times \text{that same number})$

$$= \frac{\text{Numerator} \times \text{any number}}{\text{Denominator} \times \text{that same number}}$$

(2) The given fraction $= \dfrac{\text{Numerator}}{\text{Denominator}} = \text{Numerator} \div \text{Denominator}$

$= (\text{Numerator} \div \text{any number}) \div (\text{Denominator} \div \text{that same number})$

$$= \frac{\text{Numerator} \div \text{any number}}{\text{Denominator} \div \text{that same number}}$$

Examples

$$\frac{1}{2} = \frac{1\times2}{2\times2} = \frac{2}{4} = \frac{1\times3}{2\times3} = \frac{3}{6} = \frac{1\times4}{2\times4} = \frac{4}{8} = \frac{1\times8}{2\times8} = \frac{8}{16}.$$

$$\frac{1}{3} = \frac{1\times2}{3\times2} = \frac{2}{6} = \frac{1\times3}{3\times3} = \frac{3}{9} = \frac{1\times4}{3\times4} = \frac{4}{12} = \frac{1\times9}{3\times9} = \frac{9}{27}$$

$$\frac{24}{36} = \frac{24\div2}{36\div2} = \frac{12}{18} = \frac{12\div2}{18\div2} = \frac{6}{9} = \frac{6\div3}{9\div3} = \frac{2}{3}.$$

$$\frac{15}{35} = \frac{15\div5}{35\div5} = \frac{7}{12}$$

So, from the above rules of division the new property of a fraction obtained is as follows :

The value of the fraction remains unaltered if the numerator and denominator of a fraction are each multiplied or divided by the same number.

Note : In case of division by the same, one should be careful that the number should be a common factor of numerator and denominator. The rule will also hold good if the number is not a common factor but in that case the fraction will take a complicated form in which numerator and denominator will be fractions. In that case we shall not call those fractions as 'simple fractions'.

Let us consider the following example :

$$\frac{24}{36}=\frac{12}{18}=\frac{6}{9}=\frac{2}{3}.$$

Thus the fraction is expressed in lower terms when divided by a common factor of numerator and denominator. By this repeated process of divison, when the numerator and denominator becomes relatively prime, the fraction is said to be in its lowest terms. Since H. C. F. is the greatest common factor, the fraction obtained by division of numerator and denominator by their H. C. F. is the lowest or simplest form of the given fraction.

If the numerator and denominator be resolved into factors, then the fraction can be written in the lowest term by cancelling common factors.

Example. Express $\frac{16}{36}$ in lowest terms.

$$\frac{16}{36}=\frac{2\times2\times2\times2}{2\times2\times3\times3}=\frac{4}{9}.$$

Usual practice is to write the quotient obtained by dividing numerator and denominator of a fraction by a common factor and the quotients are written by the side of respective numerator and denominator. If the quotients thus obtained again possess a common factor, above technique is applied again and again till the fraction is expressed in the lowest term.

Thus
$$\frac{\overset{\overset{4}{\cancel{8}}}{\cancel{16}}}{\underset{\underset{9}{\cancel{18}}}{\cancel{36}}}=\frac{4}{9}.$$

If any of numerator or denominator is fully exhausted by the process of cancellation, 1 is put in that place.

$$\text{Hence,}\quad \frac{\overset{\overset{1}{\cancel{5}}}{\cancel{15}}}{\underset{\underset{3}{\cancel{15}}}{\cancel{45}}} = \frac{1}{3}.$$

When it is difficult to ascertain the common factors, then we have to take help of H. C. F. of numerator and denominator to find fractions in its lowest terms.

Comparison of fractions

We first compare two fractions having same denominator. For example, take two fractions $\frac{2}{7}$ and $\frac{5}{7}$. Now $\frac{2}{7}$ means 2 parts of 7 equal divisions of a thing and $\frac{5}{7}$ means 5 parts of 7 equal divisions of the same thing. Obviously 5 parts of 7 divisions is greater than 2 parts of 7 divisions.

i. e. $\frac{5}{7} > \frac{2}{7}$ or $\frac{2}{7} < \frac{5}{7}$.

Likewise we say that

$$\frac{3}{11} < \frac{8}{11} \text{ or } \frac{7}{13} > \frac{5}{13}.$$

Now if the fractions have different denominators, then we shall have to convert them into same denominator in order to compare the fractions.

Take the example of the fractions $\frac{1}{2}$ and $\frac{1}{3}$.

We know that $\frac{1}{2} = \frac{1 \times 3}{2 \times 3} = \frac{3}{6}$

$\frac{1}{3} = \frac{1 \times 2}{3 \times 2} = \frac{2}{6}$

Now $\frac{3}{6} > \frac{2}{6}$ as $3 > 2$

Hence $\frac{1}{2} > \frac{1}{3}$.

Again, consider the fractions $\frac{5}{7}$ and $\frac{8}{11}$.

Now $\frac{5}{7}=\frac{5\times11}{7\times11}=\frac{55}{77}$.

and $\frac{8}{11}=\frac{8\times7}{11\times7}=\frac{56}{77}$.

Now $\frac{56}{77}>\frac{55}{77}$

Hence $\frac{8}{11}>\frac{5}{7}$.

To compare two fractions, the fractions are to be converted into fractions having same denominator and then the greater the numerator, the greater the fraction will be.

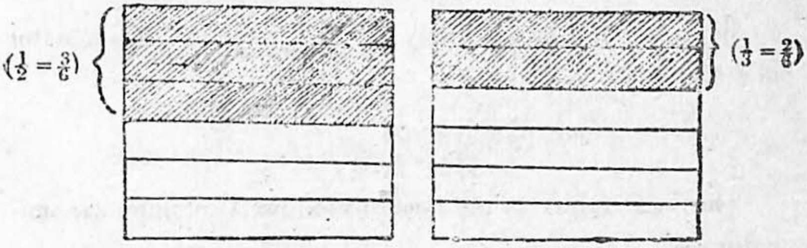


Figure indicating $\frac{1}{2}>\frac{1}{3}$

We know that we can multiply the numerator and the denominator by the same factor without changing the value of the fractional number. For example,

$$\frac{1}{2}=\frac{1\times3}{2\times3}=\frac{3}{6}$$

or $\frac{1}{2}=\frac{1\times6}{2\times6}=\frac{6}{12}$.

Again the numerator and denominator can be divided by the same factor without changing the value of the fractional number. For example,

$$\frac{3}{6}=\frac{3\div3}{6\div3}=\frac{1}{2}$$

The simplest and most convenient way of converting two or more fractions to fractions having same denominator is to use the idea of least common multiple. If the common denominators be the L. C. M. of the denominators of the fractions, then the fractions are reduced to fractions having least common denominator.

Example 1. Convert the following fractions with lowest common denominator and arrange them in ascending and descending order of magnitude :

$$\tfrac{2}{3}, \tfrac{4}{15}, \tfrac{3}{5}.$$

L. C. M. of 3, 15, 5 is 15.

Since $15 \div 3 = 5$, we multiply the numerator and denominator of $\frac{2}{3}$ by 5.

$$\therefore \quad \tfrac{2}{3} = \tfrac{2 \times 5}{3 \times 5} = \tfrac{10}{15}.$$
$$\tfrac{3}{5} = \tfrac{3 \times 3}{5 \times 3} = \tfrac{9}{15}.$$

Thus the forms of the fractions in lowest common denominator are

$$\tfrac{10}{15}, \tfrac{4}{15} \text{ and } \tfrac{9}{15}.$$

Since $4 < 9 < 10$, the fractions in the ascending order of magnitude are arranged as $\frac{4}{15} < \frac{3}{5} < \frac{2}{3}$ and in descending order of magnitude as $\frac{2}{3} > \frac{3}{5} > \frac{4}{15}$.

Example 2. Which of the following fractions is the smallest :

$$\tfrac{3}{5}, \tfrac{8}{15} \text{ and } \tfrac{16}{25}\ ?$$

L. C. M. of 5, 15, $25 = 75$,

$$75 \div 5 = 15, \text{ hence } \tfrac{3}{5} = \tfrac{3 \times 15}{5 \times 15} = \tfrac{45}{75}$$
$$75 \div 15 = 5, \text{ hence } \tfrac{8}{15} = \tfrac{8 \times 5}{15 \times 5} = \tfrac{40}{75}$$
$$75 \div 25 = 3, \quad ,, \quad \tfrac{16}{25} = \tfrac{16 \times 3}{25 \times 3} = \tfrac{48}{75}.$$

Since 40 is the lowest among 45, 40 and 48, the fraction $\frac{40}{75}$ is lowest among $\frac{45}{75}$, $\frac{40}{75}$ and $\frac{48}{75}$. Or in other words, $\frac{8}{15}$ is the smallest among $\frac{3}{5}$, $\frac{8}{15}$, $\frac{16}{25}$.

Exercise—12

[Examples 1 to 5 for oral. Answers of ex. 1 to 8 must be given in integers.]

1. How much is $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$-th of Re. 1 ?
2. How much is $\frac{1}{10}$, $\frac{2}{5}$ and $\frac{3}{25}$ th part of Rs. 5 ?
3. How many days are $\frac{2}{7}$ th part of a week ?
4. How much is $\frac{1}{5}$, $\frac{1}{8}$ th of 4 metres ?
5. What will be the $\frac{5}{12}$, $\frac{3}{20}$ and $\frac{2}{15}$ th part of 1 hour ?
6. How much is $\frac{1}{25}$, $\frac{1}{50}$, $\frac{1}{125}$ th part of 1 quintal ?
7. How much is $\frac{3}{20}$ th part of 15 litres ?
8. How much is $\frac{2}{15}$ th part of Rs. 9 ?

Fill up the gaps :

9. $\frac{1}{5}=\frac{}{15}=\frac{4}{}=\frac{}{60}$
10. $\frac{2}{3}=\frac{}{12}=\frac{18}{}=\frac{}{45}$
11. $\frac{6}{7}=\frac{}{98}=\frac{48}{}=\frac{}{105}$
12. $\frac{12}{13}=\frac{48}{}=\frac{}{91}=\frac{}{117}$

Convert into improper fractions :

13. $3\frac{1}{5}$ 14. $12\frac{8}{15}$ 15. $315\frac{3}{5}$ 16. $201\frac{1}{11}$

Convert into mixed numbers :

17. $\frac{14}{6}$ 18. $\frac{55}{13}$ 19. $\frac{175}{38}$ 20. $\frac{243}{118}$ 21. $\frac{1012}{77}$

Convert into lowest terms :

22. $\frac{575}{125}$ 23. $\frac{739}{120}$ 24. $\frac{126}{48}$ 25. $\frac{256}{1024}$
26. $\frac{462}{1617}$ 27. $\frac{312}{3900}$ 28. $\frac{3264}{9180}$

Convert the following fractions into fractions with lowest common denominator :

29. $\frac{3}{5}, \frac{1}{15}, \frac{1}{30},$
30. $\frac{1}{4}, \frac{5}{6}, \frac{7}{12}, \frac{1}{24}$
31. $\frac{2}{7}, \frac{13}{21}, \frac{4}{35}, \frac{15}{49}$
32. $\frac{1}{24}, \frac{3}{32}, \frac{5}{48}, \frac{7}{64}$

Arrange the following fractions in order of magnitude (ascending and descending) :

33. $\frac{1}{2}, \frac{3}{5}, \frac{3}{10}, \frac{21}{50}$
34. $\frac{5}{12}, \frac{3}{4}, \frac{7}{8}, \frac{6}{7}$
35. $\frac{17}{24}, \frac{31}{36}, \frac{53}{60}, \frac{65}{72}$
36. $\frac{12}{17}, \frac{29}{34}, \frac{43}{51}, \frac{94}{105}$

Find the greatest and the least fractions :

37. $\frac{7}{12}, \frac{5}{16}, \frac{17}{36}$
38. $\frac{5}{8}, \frac{6}{11}, \frac{8}{22}$
39. $\frac{6}{7}, \frac{7}{9}, \frac{16}{21}, \frac{50}{63}$
40. $\frac{15}{16}, \frac{23}{24}. \frac{35}{36}, \frac{125}{144}$

Addition of fractions

Suppose that a water-melon had been cut into 16 equal pieces. You were given two such pieces and your sister got one piece. Then you got $\frac{2}{16}$ parts and your sister got $\frac{1}{16}$ th part of the whole water-melon. Can you say now how many parts you and your sister got together? Actually both of you and your sister got 3 parts out of 16 parts of water-melon i. e. $\frac{3}{16}$ th part of it. Thus to consider how many parts you and your sister got together, actually we are adding both the parts. Hence

$$\frac{2}{16}+\frac{1}{16}=\frac{3}{16}.$$

Here we find that both the fractions have the same denominator 16 and we are just adding the numerator to get the result. Thus when two fractions having same denominator, is added, their sum is a fraction with same denominator and the numerator is the sum of the numerators of the fractions added. This rule is also applicable to more than two fractions.

Thus,

$$\frac{1}{3}+\frac{1}{3}=\frac{1+1}{3}=\frac{2}{3}$$

$$\frac{1}{5}+\frac{2}{5}+\frac{3}{5}=\frac{1+2+3}{5}=\frac{6}{5}=1\frac{1}{5}$$

$$\frac{4}{7}+\frac{5}{7}+\frac{5}{7}=\frac{4+5+5}{7}=\frac{14}{7}=2.$$

If now two fractions have different denominators then we have to convert these into fractions having lowest common denominator. Then we apply the rule as above. We have to add only the numerators of the fraction and the denominator remaining the same. This is also true for more than two fractions.

For example,

$$\frac{1}{2}+\frac{2}{3}=\frac{1\times3}{2\times3}+\frac{2\times2}{3\times2}=\frac{3}{6}+\frac{4}{6}=\frac{3+4}{6}=\frac{7}{6}=1\frac{1}{6}$$

In short, we can find the L. C. M. of 2, 3 and then write as follows

$$\frac{1}{2}+\frac{2}{3}=\frac{1\times3+2\times2}{6}=\frac{3+4}{6}=\frac{7}{6}=1\frac{1}{6}.$$

The working rule is: Find the L. C. M. of the denominators of the fractions to be added. Place the L. C. M. below a horizontal bar. The L. C. M. is then divided by each denominator and the result is multiplied by the corresponding numerator. The results thus obtained is placed above the horizontal line by putting '+' sign between them. The sum of these numbers is the numerator of the sum of the fractions and denominator is the L. C. M. of the denominators of the fractions. If the sum of the fractions is improper, it is expressed as mixed number whenever possible and the result is expressed in the lowest terms.

Example 1. Add $\frac{1}{4}+\frac{3}{8}+\frac{5}{16}$

L. C. M. of 4, 8, 16 is 16

Now $\frac{1}{4}+\frac{3}{8}+\frac{5}{16}=\frac{1\times4+3\times2+5\times1}{16}$

$=\frac{4+6+5}{16}=\frac{15}{16}.$

Example 2. Add $2\frac{3}{5}+1\frac{2}{3}+\frac{1}{6}$

In this example, we have to convert the mixed numbers to improper fraction and then add.

$$2\frac{3}{5}+1\frac{2}{3}+\frac{1}{6}=\frac{13}{5}+\frac{5}{3}+\frac{1}{6}$$
$$=\frac{78+50+5}{30}\ [\text{L. C. M. of } 5, 3, 6=30\ ]$$
$$=\frac{133}{30}=4\frac{13}{30}.$$

Alternative Method

Without converting the mixed numbers into improper fractions addition can also be done by separating integral parts of the fraction.

$$2\frac{3}{5}+1\frac{2}{3}+\frac{1}{6}$$
$$=2+\frac{3}{5}+1+\frac{2}{3}+\frac{1}{6}=2+1+\frac{3}{5}+\frac{2}{3}+\frac{1}{6}$$
$$=3+\frac{18+20+5}{30}=3+\frac{43}{40}=3+1\frac{13}{40}=4\frac{13}{40}.$$

Remark: Alternative process is helpful if the integral parts of mixed numbers are large.

Example 3. Add ; $999+3\frac{2}{5}+\frac{844}{15}+\frac{1021}{3}$

$=999+3\frac{2}{5}+56\frac{4}{15}+340\frac{1}{3}$
$=999+3+\frac{2}{5}+56+\frac{4}{15}+340\frac{1}{3}$
$=999+3+56+340+\frac{2}{5}+\frac{4}{15}-\frac{1}{3}$
$=1398+\frac{6+4+5}{15}=1398+\frac{15}{15}$
$=1398+1=1399.$

Subtraction of fractions

Method of subtraction is exactly similar to that of addition of fractions. Only '−' sign is put in place of '+' sign. Here is an example of addition and subtraction side by side.

Add : $\frac{2}{3}+\frac{1}{4}$ $\frac{2}{3}+\frac{1}{4}=\frac{8+3}{12}$ $=\frac{11}{12}$	Subtract : $\frac{2}{3}-\frac{1}{4}$ $\frac{2}{3}-\frac{1}{4}=\frac{8-3}{12}=\frac{5}{12}$

Example 1. Subtract: $2\frac{1}{3}-1\frac{1}{2}$

$2\frac{1}{3}-1\frac{1}{2}=\frac{7}{3}=\frac{3}{2}=\frac{14-9}{9}=\frac{5}{6}$

Example 2, Simplify :: $1\frac{2}{5}+2\frac{1}{3}-2\frac{1}{5}$

Given expression $=\frac{7}{5}+\frac{7}{3}-\frac{11}{5}$
$=\frac{21+35-33}{15}=\frac{56-33}{15}=\frac{23}{15}=1\frac{8}{15}$

Example 3. Simplify : $2-\frac{9}{11}+\frac{1}{33}-\frac{1}{44}$

Given expression $=\frac{2}{1}-\frac{9}{11}-\frac{1}{33}-\frac{1}{44}$
$=\frac{2\times 132-9\times 12+1\times 4-1\times 3}{132}$
$=\frac{264-108+4-3}{132}=\frac{268-111}{132}$
$=\frac{157}{132}=1\frac{25}{132}$

Exercise—13

Find the sum of :

1. $\frac{2}{3}+\frac{5}{12}$ 2. $\frac{3}{5}+\frac{4}{15}+\frac{2}{25}$ 3. $1+\frac{2}{7}+2\frac{4}{13}$

4. $\frac{15}{64}+2\frac{5}{32}+\frac{1}{48}$ 5. $\frac{1}{12}+\frac{5}{16}+\frac{7}{32}+\frac{8}{15}$

6. $2+3\frac{9}{13}+\frac{1}{26}+\frac{7}{39}$ 7. $10\frac{1}{11}+11\frac{2}{33}+9\frac{7}{44}$

8. $4\frac{1}{17}+\frac{35}{68}+\frac{3}{102}$ 9, $\frac{1}{5}+\frac{1}{6}+\frac{1}{7}+\frac{1}{8}+\frac{1}{9}$

10. $5\frac{1}{121}+3\frac{2}{11}+\frac{2}{50}$ 11. $124\frac{2}{7}+322\frac{5}{14}$

12, $339\frac{1}{10}+660\frac{9}{10}$ 13. $1+\frac{1}{2}+2+\frac{1}{3}+4+\frac{1}{5}$

14. $3\frac{3}{16}+4\frac{5}{12}+1\frac{2}{3}+2\frac{1}{8}+5$

15. $912+\frac{2}{3}+\frac{506}{6}+10\frac{13}{24}$

Find the result of Subtraction:

16. $3\frac{2}{3}-1\frac{5}{9}$ 17. $5-4\frac{1}{5}$ 18. $2\frac{3}{7}-1\frac{5}{11}$

19. $1-\frac{1}{199}$ 20. $\frac{2}{13}-\frac{1}{52}$ 21. $6\frac{1}{50}-3\frac{1}{75}$

22. $35\frac{7}{12}-25\frac{1}{6}$ 23. $188\frac{1}{11}-99\frac{1}{22}$

24. $504\frac{1}{2}-17$ 25. $6-\frac{1}{2}$

Simplify :

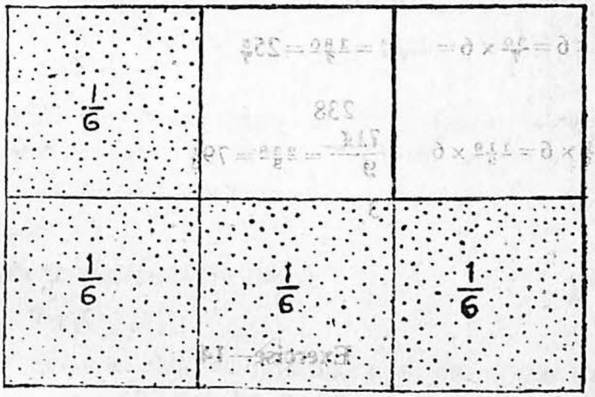
26. $1-\frac{1}{7}+\frac{1}{3}$ 27. $2\frac{2}{3}-1\frac{1}{6}-\frac{1}{3}$

28. $1-\frac{1}{2}-\frac{1}{4}-\frac{1}{8}$ 29. $4\frac{2}{15}+6\frac{1}{5}-7\frac{1}{10}$

30. $\frac{50}{51}+\frac{1}{17}-\frac{3}{85}$ 31. $10-3\frac{3}{5}+7\frac{1}{25}-11\frac{4}{35}$

Multiplication and division of fractions by whole numbers :

Multiplication



In the above figure a rectangle is divided into six equal parts. A small rectangle thus obtained is $\frac{1}{6}$ th of the whole rectangle. Such three rectangle taken together forms $\frac{3}{6}$ i. e. $\frac{1}{2}$ part of the whole. Thus we find $\frac{1}{6}\times 3=\frac{1}{2}$. Again, two small rectangles taken together form $\frac{2}{6}$ i, e. $\frac{1}{3}$rd part of the whole rectangle. Thus $\frac{1}{6}\times 2=\frac{1}{3}$.

We know that multiplication of a number by 2 implies repeated addition of the number 2 times and multiplication of a number by 3 implies repeated addition of the number 3 times. We are also following the same rule here i. e. multiplication as repeated addition.

Thus, $\frac{1}{6}+\frac{1}{6}=\frac{1+1}{6}=\frac{2}{6}=\frac{1}{3}$

$\frac{1}{6}+\frac{1}{6}+\frac{1}{6}=\frac{1+1+1}{6}=\frac{3}{6}=\frac{1}{2}$

Thus we find from the above illustrations that multiplication of fraction by a whole number means multiplication of the numerator by the whole number, while denominator remaining the same. We generally express the result of multiplication in the lowest terms if possible.

$$\frac{2}{3} \times 15 = \frac{2 \times 15}{3} = \frac{\overset{10}{\cancel{30}}}{\cancel{3}} = 10$$

$$4\frac{2}{7} \times 6 = \frac{30}{7} \times 6 = \frac{30 \times 6}{7} = \frac{180}{7} = 25\frac{5}{7}$$

$$13\frac{2}{9} \times 6 = \frac{119}{9} \times 6 = \frac{\overset{238}{\cancel{714}}}{\underset{3}{\cancel{9}}} = \frac{238}{3} = 79\frac{1}{3}$$

Exercise—14

1. Multiply $\frac{3}{4}$ by 3, 5, 12, 18.
2. Multiply $\frac{11}{12}$ by 0, 1, 4, 6, 8.
3. Multiply $5\frac{1}{8}$ by 4, 12, 16, 20,

4. There are 40 students in a class. $\frac{9}{10}$ th of the students were present in a day. Find the number of absentees on that day.

5. The population of a city is 60000. If $\frac{2}{3}$rd of the population be Hindus, $\frac{1}{4}$ th be Muslims and the rest belongs to other communities, what is the number of members of the other communities ?

6. The weight of a soap is 125 $\frac{3}{4}$ gms. How many grams will be the weight of 12 such soaps ,

7. The length of each edge of a square field is $61\frac{1}{12}$ m. If a man runs along the four edges of the field 5 times, how many kilometres will he cover ;

8. If the price of each kilogram of rice is increased by 28 paise, what excess amount will one have to pay for $625\frac{3}{4}$ kilo grams of rice ?

Division of a fraction by whole numbers

We have seen that $\frac{1}{6}\times 3=\frac{1}{2}$

Now we can write,

$$\frac{1}{2}\div 3=\frac{1}{6}=\frac{1}{2\times 3}$$

Thus when the denominator of the fraction is multiplied by 3, the fraction is divided by 3.

Let us take another example,

$$\frac{2}{27}\times 9=\frac{2}{3}$$

This can be written in the form

$$\frac{2}{3}\div 9=\frac{2}{27}=\frac{2}{3\times 9}$$

In this example also we find that when the denominator of the fraction is multiplied by 9, the fraction is divided by 9. Hence the rule is as follows :

To divide a fraction by a whole number, the denominator of the fraction is to be multiplied by that integer.

[Note : Division by zero is not admisible.]

As for example—

$$\frac{7}{9}\div 14=\frac{\cancel{7}}{9\times\underset{2}{\cancel{14}}}=\frac{1}{18}$$

$$2\frac{3}{7}\div 5=\frac{17}{7}\div 5=\frac{17}{7\times 5}=\frac{17}{35}$$

$$16\frac{2}{3}\div 4=\frac{\overset{25}{\cancel{50}}}{3\times\underset{2}{\cancel{4}}}=\frac{25}{6}=4\frac{1}{6}$$

Exercise—15

1. Divide $\frac{3}{5}$ by 1, 5, 6, 9.
2. Divide $2\frac{1}{7}$ by 2, 3, 5, 15.

3. Divide $6\frac{3}{5}$ by 3, 11, 33.

4. A man divided $\frac{7}{12}$ of his property equally among his 3 sons. What part of the property did each son get ?

5. $\frac{3}{5}$th part of a stick was divided into 4 equal parts. $\frac{4}{5}$th part of a rope of same length as that of the stick was divided into 6 equal parts. Find the smallest parts of stick and rope and find also which ts greater in length.

Multiplication of fraction by a fraction

We have already learned the multiplication and division of a fraction by a whole number. Now we have to learn multiplication of a fraction by another fraction.

Suppose, we want to multiply $\frac{2}{3}$ by $\frac{4}{7}$ i. e. we are to find the product of $\frac{2}{3}$ and $\frac{4}{7}$. We can write.

$$\frac{2}{3} \times \frac{4}{7} = \frac{4}{7} \text{ of } \frac{2}{3}$$

Since, we know that '×' and 'of' are equivalent arithmetical operations which is also clear from the literal meaning of the word 'of'. Hence if we can find the value of $\frac{4}{7}$ of $\frac{2}{3}$, that will give us the result of the product.

$$\frac{2}{3} \times \frac{4}{7}$$

To find $\frac{4}{7}$th part of a quantity, are have to divide the quantity by 7 and then multiply the result by 4. If $\frac{2}{3}$ be taken as the number.

$$\text{then } \frac{2}{3} \times \frac{4}{7} = \frac{4}{7} \text{ of } \frac{2}{3} = \frac{2}{3} \div 7 \times 4$$

Thus to multiply a fraction by another fraction, we are breaking the whole operation into two operations. First we have to multiply the result by a whole number.

Now we know that to divide a fraction by a whole number, we multiply the denominator by the whole number and to multiply a fraction by a whole number, we multiply the numerator by the whole number.

Hence, $\frac{2}{3}\times\frac{4}{7}=\frac{2}{3}\div 7\times 4$

$$=\frac{2}{3\times 7}\times 4=\frac{2\times 4}{3\times 7}=\frac{8}{21}$$

Thus the multiplication of two fractions is same as the multiplication of the numerators and that of the denominators. This rule is applicable for all the fractions for the operation of multiplication. Consider another example,

$$\frac{3}{5}\times\frac{10}{21}=\frac{3}{5}\div 21\times 10=\frac{3}{5\times 21}\times 10$$

$$=\frac{\cancel{3}\times\overset{2}{\cancel{10}}}{\cancel{5}\times\underset{7}{\cancel{21}}}=\frac{2}{7}$$

Hence, a fraction × another fraction.

$$=\frac{\text{first numerator}\times\text{second numerator}}{\text{first denominator}\times\text{second denominator}}$$

In short, $\text{fraction}\times\text{fraction}=\dfrac{\text{numerator}\times\text{numerator}}{\text{denominator}\times\text{denominator}}$

Note that the rule for multiplying a fraction by a whole number is a special case of the above rule as any whole number, for example 3 can be written as $\frac{3}{1}$.

Example :

1. $2\frac{1}{5}\times\frac{5}{22}=\frac{11}{5}\times\frac{5}{22}=\dfrac{\cancel{11}\times\cancel{5}}{\cancel{5}\times\underset{2}{\cancel{22}}}=\frac{1}{2}$

2. $$3\frac{1}{8} \times 6\frac{2}{5} = \frac{25}{8} \times \frac{32}{5} = \frac{\overset{5}{\cancel{25}} \times \overset{4}{\cancel{32}}}{\cancel{8} \times \cancel{5}} = 20$$

Division of fraction by fractions

The division of a fraction by a fraction will not be difficult at all, if we keep in view the rule and processes which have already been discussed. We have already learnt to write a fraction as an operation of division i. e.

fraction = numerator ÷ denominator.

We also know that a fraction is reduced to an equivalent fraction if the numerator and the denominator is multiplied by the same number. The rule for division of two fractions may be obtained by proper use of the above rules.

We know that

$$\begin{aligned} 5 \div 7 &= \frac{5}{7} \\ \frac{5}{6} \div \frac{7}{6} &= \frac{5 \times 6}{7 \times 6} \\ &= \frac{5 \times 6}{6 \times 7} \\ &= \frac{5}{6} \times \frac{6}{7} \end{aligned}$$

Thus division of $\frac{5}{6}$ by $\frac{7}{6}$ is same as multiplying $\frac{5}{6}$ by $\frac{6}{7}$.

The rule we have applied in particular case, is true for all cases.

Thus division of a number by a fraction is nothing but multiplication of the number by the **reciprocal** of the fraction.

[Remark : If a fraction is written by interchanging its numerator and denominator, then the second fraction is called the reciprocal of the first. Thus reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$, reciprocal of 5 is $\frac{1}{5}$.]

Thus $$\frac{8}{5} \div \frac{1}{15} = \frac{3}{\cancel{5}} \times \frac{\overset{3}{\cancel{15}}}{1} = 9$$

$$4\tfrac{1}{3}\div 2\tfrac{4}{11}=\tfrac{13}{3}\div\tfrac{26}{11}=\frac{\cancel{13}}{3}\times\frac{11}{\underset{2}{\cancel{26}}}=\tfrac{11}{6}=1\tfrac{5}{6}$$

$$3\div 2\tfrac{4}{7}=\tfrac{3}{1}\div\tfrac{18}{7}=\frac{\cancel{3}}{1}\times\frac{7}{\underset{6}{\cancel{18}}}=\tfrac{7}{6}=1\tfrac{1}{6}$$

Some rules regarding multiplication and division of fractions.

1. Always convert mixed numbers to improper fractions. Consider the whole number as a fraction whose numerator is the whole number and the denominator is 1. For example.

$$5=\tfrac{5}{1},\qquad 10=\tfrac{10}{1}$$

2. Consider 'of' as the process of multiplication.

For example,

$$\tfrac{3}{4}\text{ of }\tfrac{2}{3}=\frac{\cancel{3}}{\underset{2}{\cancel{4}}}\times\frac{\cancel{2}}{\cancel{3}}=\tfrac{1}{2}$$

If two or more fractions are connected by 'if', always perform the operation of 'of' first. When fractions are connected with 'of' sign after a '÷' sign then at the time of performing the operation of division, all the fractions connected with 'of' are to be taken as the reciprocals for multiplication.

For example,

$$\tfrac{3}{4}\div\tfrac{2}{3}\text{ of }\tfrac{1}{9}\text{ of }\tfrac{1}{8}=\tfrac{3}{4}\div\tfrac{2\times1\times1}{3\times9\times8}$$

$$=\frac{3}{\cancel{4}}\times\frac{3\times9\times\overset{4}{\cancel{8}}}{\cancel{2}\times1\times1}=81$$

3. Convert the fractions after division to mixed number.' whenever necessary.

Example 1. Multiply $1\frac{5}{32}\times 2\frac{6}{37}$

$$1\tfrac{5}{32}\times 2\tfrac{6}{37}=\frac{\cancel{37}}{\underset{2}{\cancel{32}}}\times\frac{\overset{5}{\cancel{80}}}{\cancel{37}}=\tfrac{5}{2}=2\tfrac{1}{2}$$

Example 2. Divide $4\frac{1}{16}\div 13$

$$4\tfrac{1}{16}\div 13=\tfrac{65}{16}\div\tfrac{13}{1}=\frac{\overset{5}{\cancel{65}}}{16}\times\frac{1}{\cancel{13}}=\tfrac{5}{16}$$

Example 3. Evaluate $2\frac{1}{7}\times\frac{7}{8}$ of $\frac{12}{25}$

$$\text{Given expression}=\frac{15}{7}\times\frac{7}{\underset{2}{\cancel{8}}}\text{ of }\frac{\overset{3}{\cancel{12}}}{25}$$

$$=\frac{\overset{3}{\cancel{15}}}{\cancel{7}}\times\frac{\cancel{7}\times 3}{2\times\underset{5}{\cancel{25}}}=\tfrac{9}{10}$$

Example 4. Divide $13\frac{2}{11}\div 8\frac{4}{5}$

$$13\tfrac{2}{11}\div 8\tfrac{4}{5}=\tfrac{145}{11}\div\tfrac{44}{5}=\tfrac{145}{11}\times\tfrac{5}{44}$$
$$=\tfrac{725}{484}=1\tfrac{241}{484}.$$

Example 5. Evaluate $2\frac{2}{7} \div 2\frac{1}{5}$ of $\frac{4}{7}$

Given expression $= \frac{16}{7} \div \frac{11}{5}$ of $\frac{4}{7}$

$$= \frac{16}{7} \div \frac{11 \times 4}{5 \times 7}$$

$$= \frac{\overset{4}{\cancel{16}}}{\cancel{7}} \times \frac{5 \times \cancel{7}}{11 \times \cancel{4}} = \frac{20}{11} = 1\frac{9}{11}$$

Example 6. Evaluate $6\frac{2}{3} \div 3\frac{1}{5} \div \frac{2}{3}$

$$6\frac{2}{3} \div 3\frac{1}{5} \div \frac{2}{3} = \frac{20}{3} \div \frac{16}{5} \div \frac{2}{3}$$

$$= \frac{\overset{5}{\cancel{\overset{10}{\cancel{20}}}}}{\cancel{3}} \times \frac{5}{\underset{8}{\cancel{16}}} \times \frac{\cancel{3}}{\cancel{2}} = \frac{25}{8} = 3\frac{1}{8}$$

Exercise—16

Multiply :

1, $\frac{2}{3} \times \frac{9}{20}$ 2. $3 \times \frac{2}{21}$ 3. $3\frac{1}{5} \times \frac{15}{16}$

4. $2\frac{1}{18} \times \frac{1}{37} \times 1\frac{1}{2}$ 5. $5\frac{3}{4} \times \frac{1}{46}$ 6. $7\frac{1}{32} \times \frac{96}{125}$

7. $33\frac{3}{10} \times 4\frac{6}{111}$ 8. $21\frac{3}{5} \times 9\frac{7}{12}$ 9. $192 \times 20\frac{3}{16}$

10. $3\frac{3}{5} \times \frac{2}{9} \times 6\frac{1}{4}$ 11. $\frac{15}{28} \times 3\frac{1}{2} \times \frac{4}{225}$

12. $43\frac{7}{11} \times \frac{66}{107} \times \frac{2}{3}$ 13. $403\frac{3}{4} \times \frac{2}{17} \times \frac{5}{19}$

14. $2\frac{3}{13} \times \frac{39}{40} \times \frac{50}{87}$ 15. $3\frac{1}{3} \times \frac{9}{85} \times 1\frac{8}{9} \times \frac{5}{6}$

Divide :

16. $2\frac{1}{7} \div \frac{6}{35}$
17. $8\frac{1}{3} \div \frac{20}{31}$
18. $5 \div \frac{15}{16}$
19. $3\frac{4}{17} \div \frac{11}{34}$
20. $11\frac{2}{3} \div \frac{7}{24}$
21. $5\frac{5}{28} \div 17\frac{6}{7}$
22. $70\frac{2}{3} \div 26\frac{1}{2}$
23. $45\frac{5}{6} \div 78\frac{1}{8}$
24. $200\frac{5}{11} \div \frac{17}{244}$
25. $99\frac{1}{2} \div 3\frac{1}{8}$
26. $111\frac{1}{4} \div 5\frac{9}{16}$
27- $\frac{13}{20} \div \frac{520}{112}$
28. $27\frac{3}{4} \div 14\frac{4}{5} \div 1\frac{7}{8}$
29. $3\frac{1}{5} \div 2\frac{5}{6} \div 6\frac{4}{5} \div \frac{5}{189}$

Find the value of :

30. $\frac{1}{5}$ of $1\frac{2}{3} \div \frac{1}{9}$
31. $3\frac{2}{3} \div 4\frac{7}{12}$ of $\frac{4}{5}$
32. $1\frac{1}{11} \times \frac{2}{9} \div 1\frac{7}{33}$
33. $20 \div \frac{4}{5} \div \frac{8}{15}$
34. $\frac{49}{80}$ of $\frac{4}{7}$ of $\frac{2}{3}$

Simplification involving fractions

If some fractions are connected by the signs ‘+’, ‘–’, ‘×’, ‘÷’, and ‘of’ etc. in an expression, then the simplification or such expression begins with the operation ‘of’ followed by the operations of division, multiplication, addition and subtraction. If an expression of the above type contains vinculum ‘–’ and brackets, then the simplification begins with vinculum ‘–’ followed by first bracket ( ), then second bracket { } and then third bracket [ ].

For simplification involving fractions, it is always convenient to convert the mixed numbers into improper fractions. If the terms of the expression connected by '×' and '÷' then we have to perform these operations at the same time after changing the sign of division into sign of multiplication according to the rule.

Example 1. Simplify :

$$2\frac{2}{3}\div\frac{2}{3}\text{ of }\frac{4}{5}\times\frac{1}{6}+1\frac{1}{5}-2\frac{1}{2}\times\frac{1}{4}$$

Given expression $=\frac{8}{3}\div\frac{8}{15}\times\frac{1}{6}+\frac{6}{5}-\frac{5}{2}\times\frac{1}{4}$

$$=\frac{8}{3}\div\frac{8}{15}\times\frac{1}{6}+\frac{6}{5}-\frac{5}{8}$$

$$=\frac{\cancel{8}}{\cancel{3}}\times\frac{\overset{5}{\cancel{15}}}{\cancel{8}}\times\frac{1}{6}+\frac{6}{5}-\frac{5}{8}$$

$$=\frac{5}{6}+\frac{6}{5}-\frac{5}{8}$$

$$=\frac{100+144-75}{120}=\frac{244-75}{120}$$

$$=\frac{169}{120}=1\frac{49}{120}$$

Example 2. Simplify :

$$\frac{1}{12}\times\frac{2}{3}-\frac{5}{12}\div\frac{3}{4}+\frac{9}{16}\div\frac{4}{3}\text{ of }\frac{3}{16}$$

Given expression :—

$$= \frac{1}{\underset{6}{\cancel{12}}} \times \frac{\cancel{2}}{3} - \frac{5}{\underset{3}{\cancel{12}}} \times \frac{\cancel{4}}{3} + \frac{9}{16} \div \frac{\cancel{4} \times \cancel{3}}{\cancel{3} \times \underset{4}{\cancel{16}}}$$

$$= \frac{1}{18} - \frac{5}{9} + \frac{9}{16} \div \frac{1}{4}$$

$$= \frac{1}{18} - \frac{5}{9} + \frac{9}{\underset{4}{\cancel{16}}} \times \cancel{4}$$

$$= \frac{1}{18} - \frac{5}{9} + \frac{9}{4}$$

$$= \frac{2 - 20 + 81}{36}$$

$$= \frac{\overset{7}{\cancel{63}}}{\underset{4}{\cancel{36}}} = \frac{7}{4} = 1\frac{3}{4}$$

Example 3. Simplify :

$$1\frac{2}{9} \div \left[\frac{3}{4} - 2\frac{2}{3}\left\{\frac{3}{8} - 2\frac{1}{2}\left(\frac{1}{4} - \frac{1}{6}\right)\right\}\right]$$

[Note :—Observe that second bracket has started after $2\frac{2}{3}$ and there is no sign between them. In these cases we assume that there is sign 'of' between them.]

Given exp. : $= \frac{11}{9} \div \left[\frac{3}{4} - \frac{8}{3}\left\{\frac{3}{8} - \frac{5}{2}\left(\frac{3-2}{12}\right)\right\}\right]$

$$= \frac{11}{9} \div \left[\frac{3}{4} - \frac{8}{3}\left\{\frac{3}{8} - \frac{5 \times 1}{2 \times 12}\right\}\right]$$

$$= \frac{11}{9} \div \left[\frac{3}{4} - \frac{8}{3}\left\{\frac{3}{8} - \frac{5}{24}\right\}\right]$$

$$= \frac{11}{9} \div \left[\frac{3}{4} - \frac{8}{3}\left\{\frac{9-5}{24}\right\}\right]$$

$$= \frac{11}{9} \div \left[\frac{3}{4} - \frac{\cancel{8} \times 4}{3 \times \underset{3}{\cancel{24}}}\right]$$

$$= \frac{11}{9} \div \left[\frac{3}{4} - \frac{4}{9}\right]$$

$$= \frac{11}{9} \div \left[\frac{27-16}{36}\right] = \frac{11}{9} \div \frac{11}{36}$$

$$= \frac{\cancel{11}}{\cancel{9}} \times \frac{\overset{4}{\cancel{36}}}{\cancel{11}} = 4$$

Example 4 Simplify :

$$2\tfrac{2}{5}\Big[\tfrac{3}{4}-2\tfrac{2}{3}\Big\{\tfrac{3}{8}-\tfrac{1}{2}\Big(\tfrac{1}{3}-\tfrac{1}{4}\overline{-\tfrac{1}{6}}\Big)\Big\}\Big]$$

Given exp. : $=\tfrac{12}{5}\Big[\tfrac{3}{4}-\tfrac{8}{3}\Big\{\tfrac{3}{8}-\tfrac{1}{2}\Big(\tfrac{1}{3}-\tfrac{3-2}{12}\Big)\Big\}\Big]$

$$=\tfrac{12}{5}\Big[\tfrac{3}{4}-\tfrac{8}{3}\Big\{\tfrac{3}{8}-\tfrac{1}{2}\Big(\tfrac{1}{3}-\tfrac{1}{12}\Big)\Big\{\Big]$$

$$=\tfrac{12}{5}\Big[\tfrac{3}{4}-\tfrac{8}{5}\Big\{\tfrac{3}{8}-\tfrac{1}{2}\Big(\tfrac{4-1}{12}\Big)\Big\}\Big]$$

$$=\tfrac{12}{5}\left[\;-\;\left\{\frac{3}{8}-\frac{1\times\cancel{3}}{2\times\underset{4}{\cancel{12}}}\right\}\right]$$

$$=\tfrac{12}{5}\Big]\tfrac{3}{4}-\tfrac{8}{3}\Big\{\tfrac{3}{8}-\tfrac{1}{8}\Big\}\Big]$$

$$=\tfrac{12}{5}\left[\frac{3}{4}-\frac{\cancel{8}\times 2}{3\times\cancel{8}}\right]=\tfrac{12}{5}\Big[\tfrac{3}{4}-\tfrac{2}{3}\Big]$$

$$=\tfrac{12}{5}\Big[\tfrac{9-8}{12}\Big]=\frac{\cancel{12}}{5}\times\frac{1}{\cancel{12}}=\tfrac{1}{5}$$

Example 5. Simplify :

$$\tfrac{2}{3}\div\tfrac{4}{9}\text{ of }7\tfrac{1}{2}+999\tfrac{494}{495}\times 99$$

Given exp. : $=\tfrac{2}{3}\div\tfrac{4}{9}\text{ of }\tfrac{15}{2}+\big(999+\tfrac{494}{495}\big)\times 99$

$$=\frac{\cancel{2}}{\cancel{3}}\times\frac{\cancel{2}\times\overset{3}{\cancel{9}}}{\underset{5}{\cancel{15}}\times\cancel{4}}+999\times 99+\frac{494}{\underset{5}{\cancel{495}}}\times\cancel{99}$$

$$=\tfrac{1}{5}+(1000-1)\times 99+\tfrac{494}{5}$$

$$=\tfrac{1}{5}+\tfrac{494}{5}-99000-99$$

$$=\tfrac{1+494}{5}+99000-99$$

$$=\tfrac{495}{5}+99000-99$$

$$=99+99000-99=99000.$$

Exercise—17

Simplify :—

1. $2\frac{1}{3}+\frac{7}{9}$ of $\frac{3}{8}+1-\frac{5}{24}+3\frac{1}{3}$
2. $5-\frac{3}{5}\div\frac{4}{5}$ of $\frac{3}{4}$ of $\frac{2}{3}$
3. $1\frac{3}{8}+3\frac{1}{3}-5\frac{5}{6}+2\frac{1}{4}-1\frac{1}{12}$
4. $2\frac{3}{4}-\frac{1}{3}$ of $\frac{3}{5}\times\frac{5}{8}-1\frac{3}{16}-2\frac{1}{4}\times\frac{1}{2}$
5. $\frac{1}{5}\times\frac{1}{3}\div\frac{54}{15}\div\frac{1}{2}$ of $\frac{1}{8}$
6. $215+1\frac{1}{33}$ of $\frac{1}{9}$ of $66-\frac{7}{12}\div\frac{3}{16}-\frac{1}{9}\times 4$
7. $(1+\frac{5}{6}+\frac{7}{8}+\frac{11}{12})\div(\frac{3}{4}-\frac{5}{8})$
8. $\{\frac{11}{16}\div(\frac{5}{8}+\frac{3}{4})\}-(\frac{7}{9}\div 1\frac{5}{9})$
9. $2\frac{2}{5}[\frac{3}{4}\div 1\frac{1}{3}\{\frac{3}{8}-1\frac{1}{2}(\frac{1}{4}-\frac{1}{6})\}]$
10. $1\div[2+2\div\{2+2\div(2+2\div 4)\}]$
11. $(\frac{2}{3}-\frac{3}{5})$ of $(6\frac{1}{4}-3\frac{7}{8})\div\{5-(2\frac{1}{2}-1\frac{1}{2})\}$
12. $7\frac{1}{2}-[3\frac{1}{4}\div\{\frac{3}{4}-\frac{1}{3}(\frac{2}{3}-\overline{\frac{1}{6}-\frac{1}{8}})\}]$
13. $\frac{15}{16}\div 1\frac{7}{8}[\frac{3}{2}(\frac{1}{3}+\frac{1}{5})-(\frac{7}{15}-\frac{1}{6})]$
14. $3-\{\frac{1}{3}+(2\frac{1}{2}-\overline{\frac{3}{4}+\frac{5}{6}})\}\div\frac{5}{12}$
15. $\{(6\frac{7}{8}+3\frac{4}{5})\div(6\frac{7}{8}-3\frac{4}{5})\div\frac{1}{3}$ of $10\frac{17}{41}$
16. $\frac{3}{5}[4-\frac{1}{4}\{4-\frac{2}{5}(4-\overline{\frac{1}{2}+\frac{1}{6}})\}]$
17. $[\frac{3}{4}\div\frac{3}{8}$ of $\frac{2}{3}-\{\frac{1}{2}\times\frac{3}{4}+(\frac{1}{3}-\overline{\frac{1}{6}+\frac{1}{12}})\}]$
18. $3\frac{2}{3}\div 2\frac{3}{4}-(2\frac{4}{5}\div 7)\div(\frac{2}{3}$ of $\frac{4}{5})+\frac{5}{12}$

Complex Fraction

The numerator and denominator of the fractions that we have considered so far are whole numbers. The fractions whose numerators or denominators or both are fractions are known as complex fractions. Examples are,

$$\frac{\frac{1}{3}}{5},\ \frac{2}{\frac{3}{5}},\ \frac{\frac{3}{7}}{\frac{4}{9}},\ \frac{\frac{1}{2}+\frac{1}{3}}{\frac{1}{3}+\frac{1}{4}}\ \text{etc.}$$

We use a bold line to separate the numerator and denominator of a complex fraction. The numerator and denominator of a complex fraction can be treated as dividend and divisor respectively. For example,

$$\frac{\frac{1}{3}}{5}=\frac{1}{3}\div 5=\frac{1}{3\times 5}=\frac{1}{15}$$

$$\frac{2}{\frac{3}{5}}=2\div\frac{3}{5}=2\times\frac{5}{3}=\frac{10}{3}=3\frac{1}{3}$$

$$\frac{\frac{3}{7}}{\frac{4}{9}}=\frac{3}{7}\div\frac{4}{9}=\frac{3}{7}\times\frac{9}{4}=\frac{27}{28}$$

In many cases where the numerators and denominators are expressed as the combination of several operations of fractions, the numerators and denominators are simplified first. Then the simplified results are treated as the numerator and denominator of the reduced complex fraction for further simplification.

Thus,

$$\frac{\frac{1}{2}+\frac{1}{3}}{\frac{1}{3}+\frac{1}{4}}=\frac{\frac{3+2}{6}}{\frac{4+3}{12}}=\frac{\frac{5}{6}}{\frac{7}{12}}=\frac{5}{\cancel{6}}\times\frac{\overset{2}{\cancel{12}}}{7}=\frac{10}{7}=1\frac{3}{7}$$

Sometimes each of the different parts of a complex fraction are separately simplified to get final solution easily.

Example : Simplify

$$\frac{13\frac{1}{5}}{4\frac{5}{7}} \text{ of } \frac{3\frac{1}{4}}{7\frac{4}{5}} \div \frac{2}{7}\left(2\frac{1}{11} + \frac{3}{22}\right)$$

$$\text{1st part } = \frac{13\frac{1}{5}}{4\frac{5}{7}} = \frac{\frac{66}{5}}{\frac{33}{7}} = \frac{66}{5} \div \frac{33}{7}$$

$$= \frac{\overset{2}{\cancel{66}}}{5} \times \frac{7}{\cancel{33}} = \frac{14}{5}$$

$$\text{2nd part} = \frac{3\frac{1}{4}}{7\frac{4}{5}} = \frac{\frac{13}{4}}{\frac{39}{5}} = \frac{13}{4} \div \frac{39}{5}$$

$$= \frac{13}{4} \times \frac{5}{39} = \frac{5}{12}$$

$$\text{3rd part} = \frac{2}{7}\left(2\frac{1}{11} + \frac{3}{22}\right)$$

$$= \frac{2}{7}\left(\frac{23}{11} + \frac{3}{22}\right)$$

$$= \frac{2}{7}\left(\frac{46+3}{22}\right) = \frac{2}{7} \times \frac{49}{22} = \frac{7}{11}$$

∴ The given expression

$$= \frac{14}{5} \text{ of } \frac{5}{12} \div \frac{7}{11}$$

$$= \frac{\overset{7}{\cancel{14}} \times \cancel{5}}{\cancel{5} \times \underset{6}{\cancel{12}}} \div \frac{7}{11}$$

$$= \frac{\cancel{7}}{6} \times \frac{11}{\cancel{7}} = \frac{11}{6}$$

$$= 1\frac{5}{6}$$

Look at the complex fraction given below :

$$\cfrac{1}{1+\cfrac{2}{3+\cfrac{1}{2+\frac{1}{3}}}}$$

It is worth observing the peculiarity of this fraction. It is a complex fraction with numerator as 1 and denominator as 1+another complex fraction. The numerator of the second fraction is 2 and the denominator is again 3+another complex fraction. The third fraction has numerator as 1 and denominator as $2+\frac{1}{3}$. To simplify this type of complex fractions we start from the denominator of the last complex fraction and proceed gradually upwards step by step. Fractinns of this from is called **Continued Fraction.**

Details of simplification are given below :

$$\cfrac{1}{1+\cfrac{2}{3+\cfrac{1}{2+\frac{1}{3}}}}$$

$$=\cfrac{1}{1+\cfrac{2}{3+\cfrac{1}{\cfrac{6+1}{3}}}}$$

[Simplication is done at the bottom and the same procedure has been followed at subsequent stages.]

$$=\cfrac{1}{1+\cfrac{2}{3+\cfrac{1}{\frac{7}{3}}}}$$

$$= \frac{1}{1+\dfrac{2}{3+\frac{3}{7}}}$$

$$= \frac{1}{1+\dfrac{2}{\frac{21+3}{7}}}$$

$$= \frac{1}{1+\dfrac{2}{\frac{24}{7}}}$$

$$= \frac{1}{1+\dfrac{\cancel{2}\times 7}{\underset{12}{\cancel{24}}}}$$

$$= \frac{1}{1+\frac{7}{12}}$$

$$= \frac{1}{\frac{12+7}{12}}$$

$$= \frac{1}{\frac{19}{12}}$$

$$= \frac{12}{19}$$

Take another example and try to follow, how operation of each step is being done.

Example : Simplify

$$\frac{2}{3-\frac{1}{1+\frac{2}{5}}}$$

$$\text{Given fraction} = \frac{2}{3-\frac{1}{\frac{5+2}{5}}}$$

$$= \frac{2}{3-\frac{1}{\frac{7}{5}}}$$

$$= \frac{2}{3-\frac{5}{7}}$$

$$= \frac{2}{\frac{21-5}{7}}$$

$$= \frac{2}{\frac{16}{7}}$$

$$= \frac{\cancel{2} \times 7}{\cancel{16}}$$

8

$$= \frac{7}{8}$$

Exercise—18

1. $\dfrac{4}{\frac{2}{3}}$ 2. $\dfrac{\frac{3}{4}}{12}$ 3. $27\times\dfrac{\frac{2}{15}}{3\frac{3}{5}}$ 4. $\dfrac{2+\frac{1}{2}}{3+\frac{1}{3}}$

5. $\dfrac{\frac{2}{5}+\frac{1}{5}\text{ of }\frac{1}{3}}{\frac{1}{10}(\frac{1}{3}\text{ of }\frac{2}{5})\div\frac{1}{5}}$ 6. $\dfrac{3}{\frac{2}{3}+\frac{3}{5}(1-\frac{2}{3})+3\frac{1}{3}}$

7. $\dfrac{3\frac{2}{3}}{2\frac{3}{4}}-\dfrac{2\frac{4}{5}\div 7}{\frac{4}{5}\text{ of }\frac{2}{3}}+\dfrac{85}{204}$

8. $\dfrac{2\frac{1}{5}-1\frac{2}{3}}{\frac{3}{5}+\frac{2}{3}}+\dfrac{2\frac{1}{13}}{14\frac{2}{3}}\div\dfrac{\frac{6}{11}}{7\frac{2}{9}}$

9. $9\div 7\div\dfrac{3\times\frac{4}{5}\text{ of }1\frac{1}{2}-1\frac{2}{19}}{1+\frac{1}{2}+2-\frac{1}{3}}$

10. $\dfrac{1\frac{1}{3}+\frac{3}{5}\text{ of }3\frac{1}{3}+\frac{2}{15}}{(\frac{3}{5}+\frac{1}{7})\text{ of }(2\frac{1}{2}+3\frac{1}{3})}$

11. $\dfrac{\frac{3}{4}-\frac{2}{5}}{\frac{5}{6}-\frac{2}{15}}\times\dfrac{1\frac{1}{6}}{\frac{1}{4}+\frac{1}{3}}$

12. $\dfrac{\frac{1}{2}-\frac{1}{6}}{\frac{1}{2}+\frac{1}{6}}\times\dfrac{\frac{1}{4}+\frac{1}{5}}{\frac{1}{4}+\frac{1}{8}}\div\dfrac{\frac{1}{2}\div\frac{1}{3}}{\frac{1}{2}\div\frac{1}{6}}$

13. $\dfrac{4\frac{1}{2}}{32}\times\dfrac{2\frac{2}{3}\div\frac{5}{8}}{(\frac{5}{6}\text{ of }1\frac{1}{5})\div 8\frac{1}{3}}\times\frac{2}{5}$

14. $$\frac{1}{2+\frac{1}{1+\frac{1}{4}}}$$

15. $$\frac{1}{3+\frac{1}{2+\frac{1}{7}}}$$

16. $$\frac{1}{5-\frac{2}{3-\frac{1}{3-\frac{1}{3}}}}$$

17. $$8-8\times\frac{2\frac{1}{5}-1\frac{1}{7}}{2-\frac{1}{6-\frac{1}{6}}}$$

18. $$\frac{2}{1+\frac{1}{1-\frac{1}{2}}}\times\frac{2}{\frac{3}{2}\text{ of }\frac{5}{6}\div1\frac{1}{4}}$$

19. $$\frac{3\frac{1}{5}+2\frac{7}{11}}{4\frac{7}{10}-1\frac{17}{22}}\div\frac{5}{11+\frac{7}{8+2\frac{1}{2}}}-\frac{4}{7\frac{2}{3}}$$

Solution of miscellieneous problems on vulgar fraction

To express a quantity as fraction of another quantity of same kind, one should divide the former quantity by the later and both the quantities must be expressed in the same unit.

Example 1. Express Rs. 2·75 as the fraction of Rs. 6·60

$$\text{Required fraction} = \frac{\text{Rs. } 2{\cdot}75}{\text{Rs. } 6{\cdot}60}$$

$$= \frac{\overset{\overset{5}{\cancel{55}}}{\cancel{275}}\text{ p.}}{\underset{\underset{12}{\cancel{132}}}{\cancel{660}}\text{ p}} = \tfrac{5}{12}$$

[Note the result is a pure fraction]

Example 2. What part of Rs. 1725 to be added to Rs. 125 to mak Rs. 325 ?

$$\text{Required part of money} = \text{Rs. } 325 - \text{Rs. } 125$$
$$= \text{Rs. } 200$$

Hence, the problem is to express Rs. 200 as part of Rs. 1725.

$$\therefore \text{ Required part} = \frac{\text{Rs. } 200}{\text{Rs. } 1725} = \frac{200}{1725} = \tfrac{8}{69}$$

To solve problems of fraction, 1 is taken to represent the 'whole'. Thus, to subtract a definite fraction from the whole, the fraction is subtracted from 1. For example if $\frac{1}{3}$rd part of a work is done, then the remaining part is $1 - \frac{1}{3} = \frac{2}{3}$rd.

Again if the value of a fraction is given, then the 'whole' is calculated by dividing the given value by the fraction, If the price of $\frac{2}{3}$rd part of a Sugarcane. is 60 p. then the cost of the whole Sugarcane $= 60 \text{ p.} \div \frac{2}{3} = (60 \times \frac{3}{2})\text{p.} = 90 \text{ p},$

On the other hand, If the value is given for the whole, the value of a fraction is obtained by multiplying the value of the whole by the fraction. For example, if the price of 1 bag of sugar is Rs, 600, then the price of $\frac{3}{4}$th of the bag is—

$$\text{Rs. } \overset{150}{\cancel{600}} \times \frac{3}{\cancel{4}} = \text{Rs. } 450.$$

Example 1 The colour of $\frac{1}{5}$th of a flagpole is red, $\frac{1}{3}$rd is green and that of rest is white. If the length of the part which is white is 7 m, find the length of the flagpole.

The part which is coloured white

$$= 1 - (\tfrac{1}{5} + \tfrac{1}{3}) = 1 - (\tfrac{5+3}{15})$$
$$= 1 - \tfrac{8}{15} = \tfrac{7}{15}$$

$\therefore$ The length of $\frac{7}{15}$th part of the pole $= 7\text{m}$

$\therefore$ Length of the whole pole $= 7\text{m} \div \frac{7}{15}$

$$= 7\text{m} \times \tfrac{15}{7} = 15\text{m}.$$

Example 2 A person gave $\frac{1}{3}$rd of his money to his wife and the remaining part was distributed equally among his three sons. If the wife received Rs. 2000 more than each of his sons, find the total amount of money the person had.

The part of the money wife received $= \frac{1}{3}$

Remaining part of the money $= 1 - \frac{1}{3} = \frac{2}{3}$

Each sons received $= \frac{2}{3} \div 3 = \frac{2}{3} \times \frac{1}{3} = \frac{2}{9}.$

Now $\frac{1}{3} - \frac{2}{9} =$ difference between the money of the wife and each of the son $=$ Rs. 2000.

$\frac{1}{3}-\frac{2}{9}$

$=\frac{3-2}{9}=\frac{1}{9}.$

Hence $\frac{1}{9}$th part of the money = Rs. 2000

∴ Whole = Rs. 2000 × $\frac{9}{1}$ = Rs. 18,000.

∴ The person had Rs. 18,000.

Exercise—19

1. Express first quantity as fraction of the second :

(a) Rs. 225, Rs. 375

(b) Rs. 3 and 20 p., Rs. 19 and 20 p.

(c) 3m 50 cm, 14 m.

(d) 1 litre 200 ml., 6 litre

(e) 1 quintal, 5 quintal 75 kg.

2. Evaluate the following :

(a) $1\frac{1}{4}$ of Rs. 8·20 p.

(b) $\frac{4}{5}$th of 3 kg 500 gm.

(c) $\frac{2}{3}$ of $1\frac{1}{3}$ of **Rs.** 49·77 p.

3. What amount of money taken from $\frac{2}{5}$th of Rs. 150 leaves a remainder of Rs. 30 ?

4. $\frac{5}{12}$th of certain amount of money is equal to $3\frac{3}{4}$th of Rs. 100. Find the amount of money.

5. $\frac{1}{8}$th of a bamboo pole is in mud and $\frac{1}{2}$ of it is under water. Find the part of the bamboo pole which is above water.

6. A person had Rs. 3000. He gave $\frac{2}{3}$rd of the money to his wife and $\frac{1}{5}$th part of the remaining to his daughter. He donated the rest of the money. Find the part of the money and the amount of the money he donated.

7. In a basket of mangoes, $\frac{1}{4}$th part was found to be rotten. Half of the remaining mangoes were sold leaving 75 mangoes in the basket. Find the number of mangoes the basket had originally.

8, A man spends $\frac{5}{8}$th part of his income to maintain his family, $\frac{1}{12}$th part as house rent and saves the rest. If his monthly savings is Rs. 100, find his monthly income.

9. A labourer travels $\frac{2}{3}$rd part of his journey to his factory from his house by bus, then $\frac{3}{4}$ part of the remaining by cycle and the last part on foot. If he walks a distance of 2 km, what will be the distance of the factory from his house ?

10. In a town $\frac{1}{3}$rd of total population are Muslim, $\frac{1}{4}$th are Hindus, $\frac{1}{12}$th are Christian. The remaining 1000 belongs to other religion groups. Find tne total population of the town.

11. On paying $\frac{2}{5}$th part of his debt, a man finds that he has a futher debt of Rs. 240. What was his debt ?

12. $\frac{3}{4}$the part of a reservoir was full of water. If 48 kilo litres of water was taken out for a vegetable garden, $\frac{1}{2}$ of the reservoir was still full of water. Find the total capacity of the reservoir.

13. Using fertiliser in lands, $1\frac{2}{3}$ times of paddy and $2\frac{3}{5}$ times of wheat is obtained compared to that obtained without using fertiliser. If 15 quintals of paddy are obtained from a land after using fertiliser, find the amount of paddy that could be obtained from that land without using fertiliser. In another land 10 quintals of wheat was obtained without using fertiliser. What amount of wheat could be obtained from that land after using fertiliser ?

14. A man purchased a land spending $\frac{1}{7}$th of his savings and $\frac{1}{2}$ of the rest of his savings he spent for harvesting the land. If he had then Rs. 3000 left with him, what was his total savings ?

15. A pitcher full of water weighs $16\frac{1}{2}$ kg. If $\frac{1}{4}$th part of the pitcher is full of water, the weight becomes $5\frac{1}{4}$ kg. Find the weight of the empty pitcher.

16. A book contains 120 pages. A boy read 5 pages, 6 pages and 7 pages on 1st, 2nd and 3rd day respectively. What part of the book he read in 3 days and what fraction of the whole has he yet to read ?

17. $\frac{1}{5}$th of a work was done by Ram and $\frac{1}{3}$rd of the same work was done by Karim, What part of the whole work was done by Ram and Karim both ? What part of the work was left ?

18. Through a hole of a barrel of oil, $\frac{1}{300}$th part of oil leaks out in one hour. If the barrel is full of oil, at what time half of the barrel will be empty ?

19. Kamal, Rahim and Michael take meal together. Kamal and Rahim had 3 and 2 breads respectively. Michael had no bread at all. If Michael gives 50 p. for his meal, how would Kamal and Rahim share this sum if they eat same quantity of bread ?

20. After paying Rs. 45 for a cloth a man had $\frac{2}{5}$th of the money, he took with him for shopping. What was the amount he took with him for shopping ?

21. Asit, Nihar and Arati bought a radio. Asit paid $\frac{3}{7}$th and Nihar paid $\frac{7}{20}$ th part of the cost price of the radio. What part of the cost price was paid by Arati ? If the cost of the radio was Rs. 350, then find the respective shares of Asit, Nihar and Arati.

22. Simplify :

(a) $\dfrac{\frac{2}{3} \div \frac{3}{4} \text{ of } \frac{5}{6}}{\frac{2}{3} \div \frac{3}{4} \times \frac{5}{6}} \times \dfrac{\text{4 Hectograms 5 decagrams}}{\text{5 decagrams 6grams}}$

(b) $\dfrac{\text{1 metre 92 cm}}{\text{75 cm}}$ of $\dfrac{\text{1 quintal 25 kg}}{\text{3 quintal 20 kg}}$

CHAPTER 4

Decimal fraction

We can write any number in figure with the help of ten digits namely 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. For example five hundred thirty two can be written as 532. In this number the digits 5, 3 and 2 have definite place values. Here 2 is in unit's place, 3 is in ten's place and 5 is in hundred's place. We know that the place value of unit's place is 1, that of ten's place is 10 and that of hundred's place is 100. So we can write.

$532 = 5 \times 100 + 3 \times 10 + 2 \times 1$.

Similarly, $3246 = 3 \times 1000 + 2 \times 100 + 4 \times 10 + 6 \times 1$.

The place values of the digits of a number are consecutive power of ten. The place value of a digit in the left is ten times the place value of the digits immediately to its right.

…. …	… …	Thousand's place	Hundred's place	Ten's place	Unit's place	
… …	(10000)	(1000)	(100)	(10)	(1)	
			5	3	2	$=532$
		3	2	4	6	$=3246$

It is clear that as we proceed from left to right, the place value of any digit becomes $\frac{1}{10}$th that of the previous one. For example, place value of hundred's place is $\frac{1}{10}$th of the place value of thousand's place, plaee value of ten's place is $\frac{1}{10}$th of that of hundred's place and the place value of unit's place is $\frac{1}{10}$th of that of ten's place. In the table given in page 81 there is no place to the right of unit's place. But if we proceed further one place to the right of unit's place, the place value will be $\frac{1}{10}$th of one i.e. only $\frac{1}{10}$. If we proceed another place towards the right of the place value of $\frac{1}{10}$, the place value will be $\frac{1}{10}$th of $\frac{1}{10} = \frac{1}{100}$. It appears as we go to the right side of unit's place, the place value are increasing powers of $\frac{1}{10}$. For example, $\frac{1}{10}$, $\frac{1}{10^2}$ $\frac{1}{10^3}$, and so on. Hence we can write,

←——Whole numbers→|←——Decimal part————→

Thousand's place	Hundred's place	Ten's place	Unit's place	Tenth's place	Hundredth's place	Thousandth's pluce	...	....
(1000)	(100)	(10)	(1)	($\frac{1}{10}$)	($\frac{1}{100}$)	($\frac{1}{1000}$)		

If the digits in the right of unit's place are placed according to their place values, then the names of the places may respectively be given as tenths, hundredths, thousandths etc. To seperate the digits in the right of unit's place a point is placed after the digit in unit's place. This point is known as **decimal point.**

The decimal point divides the number into two parts. The numbers on the left of decimal point are whole numbers and numbers on the right of the decimal point are known as decimal part. For example, in the number 578·215, 578 represents the whole number and 215 represents the decimal part. We read the number as "five hundred seventy eight point two one five." We never read it as "five hundred seventy eight point two hundred fifteen". If we write 578·215 in words, we write as "five hundreds 7 tens 8 ones 2 tenths one hundredths 5 thousandths."

Whole numbers	Decimal fractions
10	·35
232	·785
532	·021
30	·232
15	·00
0	·235
0	·023

Any number can be written in a decimal form.

For example, 5 = 5·000

25 = 25·000

1000 = 1000·000, etc.

It is always possible to put any number of zero's after the last digit of a decimal fraction, without changing the value of the number. For example,

3·2 = 3·2000

0·15 = 0·150

0·05 = 0·050, etc.

Observe the following table and write down the decimal fractions in the above table with the question mark (?).

Thousand's place	Hundred's place	Ten's place	Unit's place	Tenth's place	Hundredth's place	Thousandth's place	...	
			5	3	1			=5·31
5		1			2			=5010·02
				1	3	2		=0·132
	2		3			1		=203·001
			5	1	2			=15·12
	1		1		1			= ?
		2	3			4		= ?

Observe that : $5{\cdot}31=5+\frac{3}{10}+\frac{1}{100}$

$5010{\cdot}02=5010+\frac{0}{10}+\frac{2}{100}$

$0{\cdot}132=\frac{1}{10}+\frac{3}{100}+\frac{2}{1000}$ etc.

Addition and Subtraction of Decimals :

The rules for addition and subtraction for whole numbers will hold for decimal number also. For addition the decimals are to be placed so that unit's digits are under units, tenths under tenths, hundredths under hundredths and so on. The decimal point will be placed in one vertical line one under another and the point is placed in between unit's digit and tenth's digit. Now the numbers are added as it is done with whole numbers.

The decimal point is placed just below the column of decimal points.

Example : Add 4·06, 12·231 and ·025.

$$\begin{array}{r} 4{\cdot}060 \\ 12{\cdot}231 \\ {\cdot}025 \\ \hline 16{\cdot}316 \end{array}$$

In this example except the first decimal number 4·06, the other digits are upto 3 decimal places i.e. upto thousandth's place. Hence we put a zero after 6 in the first number. Thus for 4·06 we write 4·060.

Hence if any number has less number of digits to the right of the decimal point, the digits in these places are taken as zero.

For whole nnmbers, decimal point is placed as usual after unit's digit and zeros are placed after the decimal point as necessary.

Example : Add 8·292, 82, 4·12 and 0·588.

$$\begin{array}{r} 8{\cdot}292 \\ 82{\cdot}000 \\ 4{\cdot}120 \\ 0{\cdot}588 \\ \hline 95{\cdot}000 \end{array} \quad = 95$$

In this result, there are all zeros after decimal point. These zeros have no value. As such, we take 95 as the result.

For subtraction the numbers are placed vertically below one another, just as addition, so that the decimal points are in the same vertical line. The number of decimal places in the numbers are made same by placing zeros where necessary. Then we subtract, as we do with whole numbers.

For subtraction if one number is integer then the decimal point will be placed to its right and zeros may be taken after decimal point as necessary.

Example :—Subtract,

(a) 542·35—12·7 (b) 182—36·001 (c) 138·24—91

$$\text{(a)}\ \begin{array}{r} 542{\cdot}35 \\ 12{\cdot}70 \\ \hline 529{\cdot}65 \end{array} \qquad \text{(b)}\ \begin{array}{r} 182{\cdot}000 \\ 36{\cdot}001 \\ \hline 145{\cdot}999 \end{array} \qquad \text{(c)}\ \begin{array}{r} 138{\cdot}24 \\ 91{\cdot}00 \\ \hline 47{\cdot}24 \end{array}$$

We can also add or subtract decimal fractions by writing them in a row, taking into account the positions of the digits after or before the decimal point.

Example : 1. $75+1{\cdot}01=75{\cdot}00+1{\cdot}01=76{\cdot}01$

Example : 2. $85{\cdot}1-7{\cdot}441=85{\cdot}100-7{\cdot}441=77{\cdot}659$.

Remarks : In the above two examples zeros are placed after the decimal point as necessary so that both the numbers have the same number of decimal places. You can work out the addition or subtraction without placing the zeros, when you become habituated with this method. If you can practice addition or subtraction row-wise, then it will be easy for you to simplify a set of numbers containing '+' and '−' signs.

Example : $23-43{\cdot}01+65{\cdot}026-11{\cdot}432$
$=88{\cdot}026-54{\cdot}442=33{\cdot}584$.

Exercise—20

Work out the sums of this exercise, by both method one by placing the number vertically below and the other by writing the numbers in a row.

Find the Sum of :

1. $82+33{\cdot}01+3{\cdot}75+14{\cdot}8$
2. $375{\cdot}203+189{\cdot}39+892{\cdot}023+42$

3. $145{\cdot}1+309{\cdot}23+232{\cdot}832+101$
4. $3025{\cdot}23+108{\cdot}291+352{\cdot}85+92{\cdot}01$
5. $1+{\cdot}01+{\cdot}001+2+{\cdot}02+{\cdot}002$
6. $0{\cdot}325+2{\cdot}368+1{\cdot}2+0{\cdot}29$
7. $8{\cdot}35+0{\cdot}021+32{\cdot}16+39$
8. $13{\cdot}001+23{\cdot}01+0{\cdot}005+89{\cdot}6$

Find the result of Subtraction:

9. $32{\cdot}08-2{\cdot}89$
10. $95{\cdot}01-12{\cdot}183$
11. $13{\cdot}39-10{\cdot}899$
12. $3{\cdot}012-1{\cdot}089$
13. $232{\cdot}1-112{\cdot}999$
14. $333{\cdot}333-12{\cdot}29$
15. $0{\cdot}9862-0{\cdot}23$
16. $899-32{\cdot}987$
17. $525{\cdot}632-439$
18. $356{\cdot}001-3{\cdot}29$.

Simplify,

19. $2{\cdot}02-0{\cdot}35+1{\cdot}07-0{\cdot}683$
20. $1{\cdot}007+6-5{\cdot}9+2{\cdot}31$
21. $91{\cdot}02+13{\cdot}009-2{\cdot}89-50{\cdot}732$
22. $16{\cdot}02+15{\cdot}85-13{\cdot}291-15{\cdot}075$

Multiplication and Division of Decimals by 10, 100, 1000 etc.

Observe the following examples carefully :—

$$0{\cdot}2\times 10=\tfrac{2}{10}\times 10=2$$
$$0{\cdot}02\times 10=\tfrac{2}{100}\times 10=\tfrac{2}{10}=0{\cdot}2$$
$$2{\cdot}35\times 10=(2+\tfrac{3}{10}+\tfrac{5}{100})\times 10$$
$$=\tfrac{200+30+5}{100}\times 10$$

$$= \frac{235}{\cancel{100}} \times \cancel{10} = \frac{235}{10}$$

$$= 23\frac{5}{10} = 23 + \frac{5}{10} = 23\cdot5$$

(Although further cancellation is possible, it is better not to do it.)

Hence, $2\cdot35 \times 10 = 23\cdot5$

Now we find that, as a result of multiplication by 10, the decimal point is shifted by one place to the right. When the number like ·2, ·5, ·7 etc. which have only one digit after decimal point is multiplied by 10, the decimal point is placed to the right of the number. As a result the decimal point disappears and no decimal point is to be placed. Thus the result becomes $\cdot2 \times 10 = 2$, $\cdot5 \times 10 = 5$, $\cdot7 \times 10 = 7$ and so on.

Multiplication of any number by 100 is the same as multiplication of this number by 10 twice. For example,

$$\begin{aligned} 2\cdot375 \times 100 &= 2\cdot375 \times 10 \times 10 \\ &= 23\cdot75 \times 10 \\ &= 237\cdot5 \end{aligned}$$

Therefore, $2\cdot375 \times 100 = 237\cdot5$

That is, when a number is multiplied by 100 the decimal point is shifted two places to the right. Thus,

$$2\cdot75 \times 100 = 275$$

$6\cdot352 \times 100 = 635\cdot2$ and so on.

Again, $2\cdot3 \times 100 = 2\cdot3 \times 10 \times 10 = 23 \times 10 = 230$

So, $2\cdot3 \times 100 = 230$

Similarly, $0\cdot7 \times 100 = 70$

$1\cdot5 \times 100 = 150$ etc.

Hence while multiplying a decimal mumber having one digit after decimal point by 100, the decimal point should be abolished and one zero should be placed to the right of the degit. Multiplication of a number by 1000 is the same as multiplication of this number by 10 thrice :

For example, $1{\cdot}538 \times 1000 = 1{\cdot}538 \times 10 \times 10 \times 10$
$= 15{\cdot}38 \times 10 \times 10 = 153{\cdot}8 \times 10 = 1538.$

Thus, if a number is multiplied by 1000 the decimal point will be shifted three places to the right. If the number of digits in the decimal number is less than three, zeros are placed as necessary to the right of the last digit of the decimal part. For example,

$$1{\cdot}532 \times 1000 = 1532$$
$$78{\cdot}5 \times 1000 = 78500$$
$$12{\cdot}35 \times 1000 = 12350.$$

From the above discussion we find,

(i) multiplying a decimal fraction by 10, we have to shift decimal point 1 place to the right.

(ii) multiplying a decimal fraction by 100, we have to shift decimal point 2 places to the right.

(iii) multiplying a decimal fraction by 1000, we have to shift decimal point 3 places to the right and so on.

Hence, if a decimal number is multiplied by 10, 100, 1000 etc., i.e. power of 10 the decimal point will have to be moved as many places to the right as the number of zeros after 1 in the multiplier. If the number of decimal places is less than the number of zeros, zeros are to be placed to the right as necessary.

Division of Decimal fraction

We know that division is the inverse process of multiplication. Such as,

$$20 \times 2 = 40 ; \qquad 40 \div 2 = 20$$

Again, we know that

$$2{\cdot}375 \times 10 = 23{\cdot}75$$

Hence, $23{\cdot}75 \div 10 = 2{\cdot}375$

For division by 10, the decimal point is shifted one place to the left.

Again, $3{\cdot}352 \times 100 = 335{\cdot}2$

$$\therefore \quad \boxed{335{\cdot}2 \div 100 = 3{\cdot}352}$$

$$3{\cdot}5278 \times 1000 = 3527{\cdot}8$$

$$\therefore \quad \boxed{3527{\cdot}8 \div 1000 = 3{\cdot}5278}$$

So to divide a decimal fraction by 100, the decimal point will be shifted two places to the left of the decimal point and to divide by 1000 the decimal point will be shifted three places to the left and so on. That is, the decimal point will be shifted through the same number of places to the left as the number of zeros after 1 in the divisor. If the number of digits in the integral part is less than the number of zeros, then the necessary number of zeros is to be placed to the left before placing the decimal point. If the dividend is a whole number, then we will assume that the decimal point is to the right of that.

We can write,

$7 \div 10 = {\cdot}7$

$352 \div 10 = 35{\cdot}2$

$5 \div 100 = {\cdot}05$

$2{\cdot}31 \div 100 = {\cdot}0231$

$53 \div 1000 = {\cdot}053$

$45 \div 10000 = .0045$

$87 \div 10 = 8{\cdot}7$

${\cdot}1352 \div 10 = {\cdot}01352$

$13{\cdot}25 \div 100 = {\cdot}1325$

${\cdot}871 \div 100 = {\cdot}00871$

$782 \div 1000 = {\cdot}782$

$7 \div 100000 = {\cdot}00007$

Exercise—21

Find the value of :

1. (a) $4{\cdot}2 \times 10$, (b) $3{\cdot}28 \times 10$, (c) $7{\cdot}015 \times 10$.
(d) ${\cdot}003 \times 10$

2. (a) $1{\cdot}02 \times 10$, (b) $2{\cdot}35 \times 10$. (c) $8{\cdot}923 \times 10$,
(d) $0{\cdot}952 \times 10$

3. (a) ${\cdot}285 \times 100$, (b) $1{\cdot}02 \times 100$, (c) $6{\cdot}4 \times 100$,
(d) ${\cdot}8 \times 100$

4. (a) ${\cdot}004 \times 100$, (b) $1{\cdot}001 \times 100$, (c) ${\cdot}0008 \times 100$,
(d) $0{\cdot}24 \times 100$

5. (a) $1{\cdot}02 \times 1000$, (b) ${\cdot}8 \times 1000$, (c) $1{\cdot}21 \times 1000$,
(d) $3{\cdot}4 \times 1000$

6. (a) $3{\cdot}1 \times 1000$, (b) $1{\cdot}82 \times 1000$, (c) ${\cdot}021 \times 1000$,
(d) ${\cdot}55 \times 1000$

7. (a) $43 \div 10$, (b) $3{\cdot}1 \div 10$, (c) $1{\cdot}8 \div 10$,
(d) ${\cdot}4{\cdot}23 \div 10$

8. (a) $1{\cdot}41 \div 10$, (b) $3{\cdot}852 \div 10$, (c) ${\cdot}005 \div 10$,
(d) $1{\cdot}1 \div 10$

9. (a) $343 \div 100$, (b) $1{\cdot}27 \div 100$, (c) $.03 \div 100$,
(d) $1{\cdot}08 \div 100$

10. (a) ${\cdot}05 \div 100$, (b) $1{\cdot}03 \div 100$. (c) ${\cdot}08 \div 100$,
(d) ${\cdot}004 \div 100$

11. (a) 1825÷1000, (b) 618·2÷1000, (c) 1·1÷1000,
(d) ·5÷1000

12. (a) ·05÷1000, (b) 41·2÷1000, (c) ·01÷1000,
(d) 1·008÷1000

Multiplication and Division of Decimals in general

Multiplication of Decimals :

Examine the following multiplication where one is a decimal fraction and the other is a whole number.

$8\times{\cdot}2=8\times\frac{2}{10}=\frac{16}{10}=1{\cdot}6$
${\cdot}4\times7=\frac{4}{10}\times7=\frac{28}{10}=2{\cdot}8$

[According to the rule of division by 10]

Consider again the multiplication of a decimal fraction by another decimal fraction.

$1{\cdot}2\times{\cdot}13=\frac{12}{10}\times\frac{13}{100}=\frac{12\times13}{10\times100}=\frac{156}{1000}=0{\cdot}156$

[According to the rule of division by 1000]

Remarks :—Here we have written,

$1{\cdot}2=\frac{12}{10}$ and ${\cdot}13=\frac{13}{100}$

If you find difficulties to understand these, observe the following steps :

$1{\cdot}2=1+{\cdot}2=1+\frac{2}{10}=\frac{12}{10}$
${\cdot}13=\frac{1}{10}+\frac{3}{100}=\frac{10+3}{100}=\frac{13}{100}$

Similarly we can write,

${\cdot}135=\frac{135}{1000}$, $13{\cdot}5=\frac{135}{10}$,
${\cdot}0135=\frac{135}{10000}$ etc.

Again, $7{\cdot}18\times{\cdot}034=\frac{718}{100}\times\frac{34}{1000}$
$=\frac{718\times34}{100000}=\frac{24412}{100000}=0{\cdot}24412.$

From the above examples the rule for multiplication of decimal may be stated as follows :

(i) Find the product of the two decimal fractions considering them as whole number [i.e. as if there are no decimal points in the numbers].

(2) Count the number of decimal place both in the multiplier and multiplicand.

(3) Mark off in the product a number of decimal places equal to the sum of the numbers ~~of~~ decimal places in the two multiplier and multiplicand. If the number of digits in the product is less than the number of decimal places in the multiplier and multiplicand, insert zeros to the left before placing the decimal point.

Example 1.

(i) Multiply 14·21 by 0·118

(ii) Multiply 0·325 by 0·112

(i)

$$\begin{array}{r} 1421 \\ 118 \\ \hline 11368 \\ 1421\times \\ 1421\times \\ \hline 167678 \end{array}$$

$\therefore$ 14·21 × 0·118 = 1·67678.

(ii)

$$\begin{array}{r} 325 \\ \mathbf{112} \\ \hline 650 \\ 325\times \\ 325\times \\ \hline 36400 \end{array}$$

$\therefore$ 0·325 × 0·112 = ·036400

= ·0364

Exercise—22

Find the products :

1. $6{\cdot}25 \times 15$
2. ${\cdot}318 \times 26$
3. $1{\cdot}08 \times {\cdot}23$
4. $342{\cdot}1 \times 0{\cdot}34$
5. $615 \times {\cdot}302$
6. $11{\cdot}08 \times 12{\cdot}1$
7. $12{\cdot}07 \times {\cdot}051$
8. $35{\cdot}23 \times {\cdot}113$
9. $13{\cdot}001 \times 30{\cdot}4$
10. $132{\cdot}02 \times {\cdot}115$
11. $238{\cdot}1 \times {\cdot}125$
12. $116{\cdot}4 \times {\cdot}105$
13. $2{\cdot}37 \times 1{\cdot}008$
14. $223{\cdot}4 \times 1{\cdot}285$
15. $7{\cdot}58 \times {\cdot}0118$
16. $10{\cdot}09 \times {\cdot}021$
17. $83{\cdot}12 \times {\cdot}654$
18. $18{\cdot}84 \times {\cdot}132$
19. ${\cdot}33 \times {\cdot}02 \times {\cdot}18$
20. $1{\cdot}19 \times 1{\cdot}1 \times {\cdot}5$
21. $3{\cdot}01 \times {\cdot}18 \times 1{\cdot}2$

Division of Decimals :

We know that any number of zeros can be placed after the last digit in the decimal part. So we can write,

$$5 = 5{\cdot}000$$
$$2{\cdot}18 = 2{\cdot}1800$$
$$0{\cdot}8 = {\cdot}8000$$

This rule is of great importance for the division of decimals.

Let us consider first where both dividend and divisor are whole numbers. If we want to divide 15 by 4, then

$$15 \div 4 = 3\tfrac{3}{4}$$

Now, using the idea of decimals we can perform the operation of division as given below :

```
4) 15·00 ( 3·75
   12
  -----
    30
    28
   ----
     20
     20
   ----
```

Remark : $3\frac{3}{4}$ and 3·75 are same but expressed in different forms.

$$3{\cdot}75=\frac{\overset{15}{\cancel{375}}}{\underset{4}{\cancel{100}}}=\frac{15}{4}=3\frac{3}{4}$$

Now we can state the rule of division as given below,

Place a decimal point on the right of quotient when there is a remainder and then place a zero in the right of the remainder. Carry on the division so long there remains no remainder. Cases may arise when the remainder will not be exhausted. They will be discussed afterwards.

Example 1. Divide 125 by 8 applying the rule for division of decimals.

```
8 ) 125 ( 15·625
    8
   ----
    45
    40
   ----
     50
     48
    ----
      20
      16
     ----
       40
       40
      ----
```

Hence; $125 \div 8 = 15{\cdot}625$

Now the process is: we divide as in the case of whole numbers placing the decimal point in the quotient as soon as the division of the integral part is finished. Then we have to bring down the digits of the decimal part one after another and carry on the division. Even then, if there is a remainder after division, we place a zero to the right of the remainder and divide. We continue the process until there is no remainder. Cases may arise when the remainder is not exhausted. Those cases will be considered later.

Example 2. Divide 132·85 by 25

$$
\begin{array}{l}
25\ )\ 132{\cdot}85\ (\ 5{\cdot}314 \\
\quad\ \ 125 \\
\hline
\quad\quad\ \ 78 \\
\quad\quad\ \ 75 \\
\hline
\quad\quad\quad 35 \\
\quad\quad\quad 25 \\
\hline
\quad\quad\quad 100 \\
\quad\quad\quad 100 \\
\hline
\end{array}
$$

The required quotient = 5·314.

Example 3. Divide 10·32 by 40

$$
\begin{array}{l}
40\ )\ 10{\cdot}32\ (\ 0{\cdot}258 \\
\quad\quad\ 8\,0 \\
\hline
\quad\quad 232 \\
\quad\quad 200 \\
\hline
\quad\quad\ \ 320 \\
\quad\quad\ \ 320 \\
\hline
\end{array}
$$

∴ The required quotient = 0·258.

Example 4. Divide 0·582 by 15.

15) 0·582 (0·0388
45
―――
132
120
―――
120
120
―――

∴ The required quotient = 0·0388

If the divisor is also a decimal, make the divisor a whole number by shifting the decimal point to the right. Shift the decimal point in the dividend the same number of places to the right as you have shifted in the divisor. Then perform the operation of division.

To move the decimal points of dividend and divisor through the same places in the right means multiplication by the same number which may be 10 or 100 or 1000 and so on; i.e. powers of 10. We already know that the quotient remains the same if the divisor and dividend be multiplied by the same number. If the number of digits in the decimal part of the divisor is greater than that of dividend, zeros are affixed after the last digit of the decimal part of the dividend and the decimal point is removed. Then perform the operation of division.

Example 5. Divide 3·15÷2·5

$$\frac{3\cdot15}{2\cdot5}=\frac{31\cdot5}{25}$$

Now 25) 31·5 (1·26
25
―――
65
50
―――
150
150 ∴ Required quotient = 1·26
―――

Example 6. Divide 34·3 ÷ ·056

$$\frac{34{\cdot}3}{{\cdot}056} = \frac{34300}{56}$$

$$\begin{array}{r} 56\ )\ 34300\ (\ 612{\cdot}5 \\ \underline{336}\phantom{\ (\ 612{\cdot}5} \\ 70\phantom{\ (\ 612{\cdot}5} \\ \underline{56}\phantom{\ (\ 612{\cdot}5} \\ 140\phantom{\ (\ 612{\cdot}5} \\ \underline{112}\phantom{\ (\ 612{\cdot}5} \\ 280\phantom{\ (\ 612{\cdot}5} \\ \underline{280}\phantom{\ (\ 612{\cdot}5} \end{array}$$

∴ The required quotient = 612·5

Example 7. Divide 6·4 by 1248 upto 3 decimal places.

$$\begin{array}{r} 1248\ )\ 6{\cdot}400\ (\ 0{\cdot}005 \\ \underline{6{\cdot}240}\phantom{\ (\ 0{\cdot}005} \\ 160\phantom{\ (\ 0{\cdot}005} \end{array}$$

∴ The required quotient = 0·005

Remark: (1) The position of the decimal point in the quotient and placing of zeros after the decimal point are to be carefully observed. In Ex. 7 the integral part of dividend is less than that of divisor, so division of integral part is not possible and integral part of quotient is 0. The dividend becomes 64 if we consider the first digit 4 in the decimal part. Now we have to place the decimal point after zero in the quotient, as we have considered the digit 4 in the decimal part of the dividend. Now we can not divide 64 by 1248 and as such we place a 'zero' after the decimal point in the quotient. Considering the second digit in the decimal part of dividend, which is zero, we have 640 as dividend. In this case also, 640 being less than 1248, division is not possible. So one more zero is placed in the quotient and altogether we have two zeros after the decimal point in the quotient. Taking the third digit in the decimal part of the dividend, we have 6400, which is greater than divisor. So division is possible and we have third digit in the decimal part of quotient as 5. As the division is to be performed upto three places of decimal we need not procced further.

(2) Here is a case where the remainder will never be exhausted. So we have to carry on the division upto some specified places of decimal.

(3) If the divison is carried on until there is no remainder, in that case also we need not continue further, if the number of decimal places required in the quotient is already obtained.

Exercise—23

Perform the operation of division in the following cases :

1. 688·5÷17
2. 144·96÷48
3. 4018·75÷125
4. 13·608÷1·2
5. 1·554÷84
6. ·468÷1·04
7. 32·634÷31·08
8. 14·529÷125·25
9. 872·96÷682
10. 1·092÷1·04
11. 816÷·004
12. 126÷·225
13. 1·218÷14·5
14. 5·439÷222
15. 14·19÷·044
16. 14·19÷322·37 (upto three decimal places)
17. 3·584÷·035
18, 1·359÷·075
19. 33·552÷1·8
20. 9·177÷1.07 (upto three decimal places)

To convert vulgar fractions into decimal fractions

We have already learnt the division of a decimal fraction by powers of 10 e.g. 10, 100, 1000 etc. by shifting the decimal point of the dividend. For example,

$7 \div 10 = \cdot 7$, $33 \div 100 = \cdot 33$, $261 \div 1000 = \cdot 261$ etc.

Thus we can write,

$\frac{7}{10} = \cdot 7$, $\frac{33}{100} = \cdot 33$, $\frac{261}{1000} = \cdot 261$ etc.

Hence, a vulgar fraction having different powers of 10 as denominator can be converted into decimal by applying the following rule.

(1) Count the number of zeros after 1 in the denominator.

(2) Place the decimal point in the numerator leaving as many digits from the right as there are zeros in the denominator and the number thus obtained will be the decimal fraction equivalent to the vulgar fraction.

(3) If the number of digits in numerator is less than the number of zeros in denominator, place necessary number of zeros at the left of the numerator before placing the decimal point.

Usual practice for conversion of vulgar fraction into decimal is to divide the numerator by the denominator.

Example 1. Convert $\frac{12}{25}$ into decimal fraction.

$$\begin{array}{r l}
25\,)\;120\;(\,0\cdot 48 & \\
100 & \\
\hline
200 & \\
200 & \\
\hline
\end{array}$$

Hence $\frac{12}{25} = 0\cdot 48$

Example 2. Convert $2\frac{13}{16}$ into decimal fraction.

$$2\tfrac{13}{16}=2+\tfrac{13}{16}$$

$$\text{Now}\quad 16\,)\; \begin{array}{r} 130 \\ 128 \\ \hline 20 \\ 16 \\ \hline 40 \\ 32 \\ \hline 80 \\ 80 \\ \hline \end{array} \;(\,0{\cdot}8125$$

$$\therefore 2\tfrac{13}{16}=2+0{\cdot}8125=2{\cdot}8125$$

Terminating and Non-terminating decimals

In the above examples, we have seen that to convert fraction into decimals, we divide the numerator by denominator. If in the process of division, remainder becomes zero and the quotient have finite number of digits after the decimal point, then the decimal is known as **terminating decimals.**

For example,

$$\tfrac{12}{25}=0{\cdot}48, \quad 2\tfrac{13}{16}=2{\cdot}8125$$

But there are some cases where you cannot complete the division and the remainder will not be exhausted. For example $\frac{1}{3}={\cdot}333......$

Such decimal fractions in which you can not get a finite number of digits in the decimal part are called **non terminating decimals**; for examples, $\frac{1}{6}={\cdot}166....$, $\frac{2}{9}={\cdot}222...$

Now look at the following examples carefully.

$\frac{1}{2}=\cdot5$

$\frac{6}{15}=\cdot4$

$\frac{3}{8}=\cdot375$

$\frac{9}{16}=\cdot5625$

$\frac{33}{125}=\cdot264$

$\frac{231}{704}=\cdot328125$

$\frac{3}{50}=\cdot06$

$\frac{1}{3}=\cdot333......$

$\frac{2}{9}=\cdot222......$

$\frac{4}{48}=\cdot08333.......$

$\frac{18}{33}=\cdot5454.......$

$\frac{5}{84}=\cdot05952380952380.......$

$\frac{1}{7}=\cdot142857142857.......$

$\frac{321}{65}=4\cdot9384615384615....$

Observe the vulgar fractions in the left hand column. The denominators of each of the fraction, when expressed in their lowest term, have prime fractors either 2, 5 or both 2 and 5. In these cases where the denominator of the fractions have a prime factor, 2, 5 or both 2 and 5, the fractions converted into decimals will be always terminating decimals.

But on the other hand, when the denominators of the fractions on the right hand column are expressed after converting the fraction in the their lowest form as the product of its prime factors, one of the prime factors will be a different prime number than 2 and 5 (2 and 5 may be there as prime factors or not). If we divide the numerator of these fractions by denominator, we will not get a finite number of places in the decimal part. These vulgar fractions are equivalent to non-terminating decimal fractions.

Recurring decimal

Try to analyse the non-terminating decimals in the right hand column, given above. Some of the digits in the decimal part of some of the fractions when converted to decimal fractions, have recurred over and over again. For example, we can write $\frac{1}{3}=.3333....$ Here we find that the digit 3 has recurred over and over again. In case of $\frac{4}{48}=.08333...$after two places of decimal, that is, after .08, the digit 3 has recurred over and over again.

In $\frac{18}{33} = \cdot5454......$, the two digits 54 have recurred in order over and over again. Similarly in the decimal fraction equivalent to $\frac{1}{7}$, the same number of digits 142857 have recurred in order over and over again. These decimals are called recurring decimals and we write them as followe :

$\frac{1}{3} = \cdot\dot{3}$, $\frac{1}{12} = \cdot08\dot{3}$, $\frac{6}{11} = \cdot\dot{5}\dot{4}$, $\frac{1}{7} = \cdot\dot{1}4285\dot{7}$.

The whole body of the digit which recur is called the perioa and in writing a recurring decimal, we write period once and place dots over its first and last digit.

If you observe carefully then you will find that in the recurring decimals $\cdot\dot{3}$, $\cdot\dot{5}\dot{4}$, $\dot{1}4285\dot{7}$, the period has started just after the decimal point. Such recurring decimals are called pure recurring decimals. The vulgar fractions equivalent to these recurring decimals in their lowest terms are $\frac{1}{3}$, $\frac{6}{11}$, $\frac{1}{7}$ respecively. Note that 2 or 5 are not prime factors of these denominators. Hence, the vulgar fractions whose denominators when expressed in their lowest terms, have prime factors other than 2 or 5, will be equivalent to pure **recurring decimals**.

Again, observe that in the recurring decima's $\cdot1\dot{6}$, $\cdot08\dot{3}$, $\cdot05\dot{9}5238\dot{0}$, $4\cdot9\dot{3}8461\dot{5}$, the period starts after some digits in the right of the decimal point. Such recurring decimals are called mixed recurring decimals. The vulgar fractions, equivalent to these recurring decimals, when expressed in their lowest terms are $\frac{1}{6}$, $\frac{1}{12}$, $\frac{5}{84}$, $\frac{321}{65}$. The denominators of these fractions have prime factors 2 or 5 along with other prime factors. So it can be said that these fractions whose denominators have prime factors 2 or 5 along with other prime factors, when the fractions are expressed in their lowest terms are equivalent to **mixed recurring decimals**.

(1) Amalgan fraction will be a recurring decimal if it is a nonterminating decimal fraction.

(2) In finding out the recurring part will stop the division process as soon as we get one such remainder which has been obtained before.

Exercise—24

1. Convert into decimal fractions :

(a) $\frac{125}{10}$ (b) $\frac{215}{100}$ (c) $\frac{375}{1000}$ (d) $\frac{15}{1000}$
(e) $\frac{3}{1000}$ (f) $\frac{16}{125}$ (g) $3\frac{9}{16}$ (h) $4\frac{1}{400}$
(i) $\frac{524}{625}$ (j) $\frac{256}{250}$.

2. Convert into recurring decimals :

wh (a) $\frac{2}{3}$ (b) $\frac{5}{6}$ (c) $3\frac{5}{18}$ (d) $4\frac{3}{11}$ (e) $\frac{3}{7}$ (f) $\frac{5}{7}$ (g) $3\frac{5}{9}$
na (h) $\frac{8}{55}$ (i) $\frac{7}{30}$

(a) $\frac{3}{4}$ (b) $\frac{7}{9}$ (c) $3\frac{8}{24}$ (d) $\frac{7}{175}$ (e) $\frac{12}{33}$ (f) $\frac{5}{24}$
(g) $\frac{9}{360}$ (h) $\frac{17}{45}$ (i) $8\frac{9}{13}$ (j) $10\frac{10}{12}$
(k) $3\frac{18}{450}$ (l) $\frac{7}{22}$.

3. State without dividing the numerator by the denominator whether the following fractions will be converted into terminating or recurring decimals.

4. Which of the above fractions of the example 3 will be equivalent to pure recurring decimals and which will be equivalent to mixed recurring decimals ?

Simplification of decimal fractions

Example 1. Simplify :

$$3[12{\cdot}25-8\{{\cdot}65\times{\cdot}2+(1{\cdot}2-\overline{{\cdot}35-{\cdot}28})\}]$$

Given expression $=3[12{\cdot}25-8\{{\cdot}65\times{\cdot}2+(1{\cdot}2-{\cdot}07)\}]$

$$=3[12{\cdot}25-8({\cdot}65\times{\cdot}2+1{\cdot}13)]$$
$$=3[12{\cdot}25-8({\cdot}13+1{\cdot}13)]$$
$$=3[12{\cdot}25-8\times 1{\cdot}26]$$
$$=3[12{\cdot}25-10{\cdot}08]$$
$$=3[2{\cdot}17]=6{\cdot}51$$

Example 2. Simplify :

$$\frac{\cdot16+\cdot8}{\cdot84-\cdot6}+\frac{\cdot75\times6}{8\div24}-\frac{9\cdot4}{3\cdot6+1\cdot1}$$

$$\text{1st part}=\frac{\cdot16+\cdot8}{\cdot84-\cdot6}=\frac{\cdot96}{\cdot24}=\tfrac{96}{24}=4$$

$$\text{2nd part}=\frac{\cdot75\times6}{8\div24}=\frac{4\cdot50}{\frac{8}{24}}=\frac{4\cdot5}{\frac{1}{3}}$$

$$=4\cdot5\times3=13\cdot5$$

$$\text{3rd part}=\frac{9\cdot4}{3\cdot6+1\cdot1}=\frac{9\cdot4}{4\cdot7}=\frac{94}{47}=2$$

$\therefore$ Given expression $=4+13\cdot5-2=15\cdot5$

Example 3. Simplify :

$$\frac{6\cdot6\times6\cdot6-3\cdot4\times3\cdot4}{6\cdot6-3\cdot4}$$

$$\text{Given expression}=\frac{43\cdot56-11\cdot56}{3\cdot2}=\frac{32}{3\cdot2}$$

$$=\tfrac{320}{32}=10.$$

Exercise—25

Simplify :

1. $\cdot 02\times 2+\cdot 004\times 1\cdot 2+\cdot 8\div 5$

2. $3\cdot 15\times 1\cdot 05+\cdot 48\div 16+3\cdot 2\times 5$

3. $\cdot 84\times \cdot 2\div 14\times \cdot 08\div 3\cdot 2$

4. $\cdot 75\times \cdot 75+\cdot 25\times \cdot 25-2\times \cdot 75\times \cdot 25$

5. $8\cdot 5-[6\cdot 4-\{3\cdot 2-(2\cdot 5-\overline{2\cdot 1-1\cdot 8})\}]$

6. $2[16\cdot 05-\{8\cdot 3-(11\cdot 1-\overline{9\cdot 65-1\cdot 75})\}]$

7. $\dfrac{6\cdot 5\times 6\cdot 5-3\cdot 5\times 3\cdot 5}{6\cdot 5-3\cdot 5}$

8. $\dfrac{\cdot 001\times 1\cdot 44}{\cdot 016\times 3}$

9. $\dfrac{8\cdot 92-7\cdot 48}{12\cdot 36-10\cdot 56}$

10. $\dfrac{\cdot 49\times \cdot 034\times \cdot 72}{11\cdot 9\times 1\cdot 8\times \cdot 014}$

11. $\dfrac{\cdot 1701\div 16\cdot 2}{\cdot 005\times \cdot 04}$

12. $\dfrac{35\cdot 4\div \cdot 03\times \cdot 6}{\cdot 48\times 125}$

13. $(\cdot 818\times \cdot 15)+\dfrac{\cdot 009\times 1\cdot 2}{2\cdot 16}$

14. $\dfrac{\cdot 06+0\cdot 5}{\cdot 94-0\cdot 8}+\dfrac{2\cdot 5\times 6}{9\div 18}-\dfrac{6\cdot 8}{3\cdot 2\times 0\cdot 2}$

Solution of miscelleneons problems on decimal

Example 1.

(a) Express Rs 3 as a decimal of Rs. 25

$$\frac{\text{Rs. } 3}{\text{Rs. } 25}=\frac{3}{25}=\cdot 12$$

(b) Express 5 cm as a decimal of 1 metre.

$$\frac{5\text{ cm}}{1\text{ m}}=\frac{5\text{cm}}{100\text{ cm}}=\frac{5}{100}=\cdot 05$$

Example 2.

How many pens can be purchased for Rs. 60. if each pen costs Rs. 2·40 p.

Number of pens$=60\div 2{\cdot}40$,

$$=\frac{60}{2{\cdot}40}=\frac{\overset{\overset{25}{\cancel{100}}}{\cancel{600}}}{\underset{\cancel{4}}{\cancel{24}}}=25$$

Hence, 25 pens can be purchased for Rs. 60.

Exercise—26

1. If one exercise book costs Rs. 1.25 find the cost of 12 such exercise books.

Find also the number of exercise books available for Rs. 23.

2. A man went to market with Rs. 70. He purchased 4 kg. of rice at the rate of Rs. 8.10 per kg., 3 kg. of pulse at. the rate of Rs. 3.95 per kg and 1 kg. of sugar at the rate of Rs. 13.65

per kg. How much money did he spend and how much was left with him?

3. Express 200 metres as decimal of 5 kms.

4. Add : $\frac{3}{10}$ of Rs. 5, $\frac{2}{3}$ of Rs. 1·80 and $\frac{5}{12}$ of Rs. 1·92.

5. Find the time taken by a car to travel a distance of 326·8 km. if the speed of the car is 40·85 km per hour.

6. A box contains 12 packets, each packet weighing 2·84 kg. If the total weight of the box and the packets be 36 kg, find the weight of box only.

7. Weights of 4 packets are 1 kg 335 gm, 4.kg 875 gm, 2 kg 125 gm and 3 kg 222 gm respectively. Find total weight of the 4 packets.

8. What must be added to 0·685 of 325 to make it 300 ?

9. 0·15 parts of a pole of length 20 metres was under mud and 0·65 part of it was under water. Find in metre the length of the pole above water ?

10. A man has Rs. 3375 after giving 375 part of his money to his wife and ·4 part to his son. How much money did he have. How much money did his wife receive ?

11. A man pays ·15 part of his income as income-tax. He spends ·8 part of the remaining to maintain his family and the rest he saves. Find the monthly income if his savings per month is Rs. 255.

12. The circumferences of two wheels are 1·02 m and 861 m respectively. If the numbers of revolutions per minute for the first wheel be 40 and that of the second wheel be 60.

find how many metres in excess or short will the first wheel move compared to the second wheel.

13. The price of ·75 part of a property is Rs. 1800. What will be the price of $\frac{4{\cdot}8 \times {\cdot}005}{{\cdot}16}$ part of the property ?

14. There are 700 boys and 500 girls in a school. .5 part of boys has increased and .4 part of girls has decreased. What fraction of the total number of students has increased or decreased ? Express the result in decimals.

—=—

CHAPTER 5

Square root

If a number be multiplied by the same number. we say, the product is the square of the number and the number itself is termed as the square root of the product.

For example,

Square of $4=4\times4$ $=16$	Square root of $16=4$
Square of $5=5\times5$ $=25$	Square root of $25=5$
Square of $10=10\times10$ 100	Square root of $100=10$
Square of $12=12\times12$ $=144$	Square root of $144=12$ etc.

In Short, we write the square of a number, by writing 2 in short size at the top of the right hand corner of the number.

For example.

Square of $3=3\times3=3^2$

Square of $5=5\times5=5^2$

We read the square of a number as “three to the power 2”, or “five to the power 2” etc.

The symbol for square root is ‘ √ ’ and the number is written within this symbol e.g,

$$\sqrt{4}=2, \quad \sqrt{25}=5, \quad \sqrt{36}=6, \text{ etc.}$$

We read this as ‘square root of 4’ is equal to two, ‘square root of 25’ is 5 and so on.

Squares of numbers from I to 20 are tabulated below. You have to keep these results in mind.

$1^2=1$

$2^2=4$

$3^2=9$

$4^2=16$

$5^2=25$

$6^2=36$

$7^2=49$

$8^2=64$

$9^2=81$

$10^2=100$

$11^2=121$

$12^2=144$

$13^2=169$

$14^2=196$

$15^2=225$

$16^2=256$

$17^2=289$

$18^2=324$

$19^2=361$

$20^2=400$

From the above table, it is clear that :

$\sqrt{1}=1$

$\sqrt{4}=2$

$\sqrt{9}=3$

$\sqrt{16}=4$

$\sqrt{25}=5$

$\sqrt{36}=6$

$\sqrt{49}=7$

$\sqrt{64}=8$

$\sqrt{81}=9$

$\sqrt{100}=10.$

Thus we find, out of numbers from 1 to 100, we can only find the square root of 10 numbers (1, 4, 9, 16, 25, 36, 49, 64, 81, 100). Square root of the other numbers are not whole numbers. You will learn the process of finding the square root of such numbers later.

The numbers 1, 4, 25, 100 etc. which can be expressed as the sqaures of some numbers are termed as perfect squares.

For finding square root of any number in this chapter we shall always assume that the number is a whole number and a perfect square, i.e. it can be expressed as the product of two same whole numbers.

You can verify the fact that all the numbers having 2, 3, 7 or 8 in the unit place are not perfect squares. This can also be seen from the above square root table.

To determine square root

The square root can easily be found by resolving into factors.

$$4=2\times 2, \quad \therefore \sqrt{4}=2$$
$$16=2\times 2\times 2\times 2=4\times 4 \quad \therefore \sqrt{16}=4$$
$$100=2\times 2\times 5\times 5=10\times 10, \quad \therefore \sqrt{100}=10$$

Example 1. Find the square root of (a) 144 ; (b) 324

(a)
$$\begin{array}{r|l} 2 & 144 \\ \hline 2 & 72 \\ \hline 2 & 36 \\ \hline 2 & 18 \\ \hline 3 & 9 \\ \hline & 3 \end{array}$$

$$\therefore \; 144=2\times 2\times 2\times 2\times 3\times 3$$
$$=(2\times 2\times 3)\times(2\times 2\times 3)$$
$$=12\times 12=12^2$$

$$\therefore \sqrt{144}=12$$

(b)

$$\begin{array}{r|l} 2 & 324 \\ \hline 2 & 162 \\ \hline 3 & 81 \\ \hline 3 & 27 \\ \hline 3 & 9 \\ \hline & 3 \end{array}$$

$$\begin{aligned} \therefore\ 324 &= 2\times2\times3\times3\times3\times3 \\ &= (2\times3\times3)\times(2\times3\times3) \\ &= 18\times18=18^2 \end{aligned}$$

$$\therefore \sqrt{324}=18$$

In the above examples, we find that the numbers whose square roots are to be determined, can be expressed as product of prime factors and these prime factors appear in pairs or an even number of times. On the other hand the square root of that number contains those prime factors just half the time as in original number.

Thus, rule for finding square root is to resolve the numbers into prime factors and arrange them pair-wise. Then one factor from each pair is taken and the product of these factors gives the square root of the given number. By applying the above rule, the square root of 324 will be as follows.

$$324=2\times2\times3\times3\times3\times3$$

$$\sqrt{324}=2\times3\times3=18$$

We can also find the square root of 324 by writing as below.

$$324=2\times2\times3\times3\times3\times3$$

$$=2^2\times3^4$$

Hence $\sqrt{324}=2\times3^2=2\times9=18$

Example 2. Find the square root of

(a) 1296 ; (b) 28224

(a)
$$\begin{array}{r|l} 2 & 1296 \\ \hline 2 & 648 \\ \hline 2 & 324 \\ \hline 2 & 1\ 2 \\ \hline 3 & 81 \\ \hline 3 & 27 \\ \hline 3 & 9 \\ \hline & 3 \end{array}$$

$$\therefore \quad 1296 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3$$
$$= 2^2 \times 2^2 \times 3^2 \times 3^2$$
$$\sqrt{1296} = 2 \times 2 \times 3 \times 3$$
$$= 36$$

(b)
$$\begin{array}{r|l} 2 & 28224 \\ \hline 2 & 14112 \\ \hline 2 & 7056 \\ \hline 2 & 3528 \\ \hline 2 & 1764 \\ \hline 2 & 882 \\ \hline 3 & 441 \\ \hline 3 & 47 \\ \hline 7 & 49 \\ \hline & 7 \end{array}$$

$$\therefore \quad 28224 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 7 \times 7$$
$$= 2^2 \times 2^2 \times 2^2 \times 3^2 \times 7^2$$
$$\therefore \quad \sqrt{28224} = 2 \times 2 \times 2 \times - \times 7$$
$$= 2^3 \times 3 \times 7$$
$$= 8 \times 3 \times 7$$
$$= 168$$

Example—3. Find the least whole number by which 1200 must be multiplied in order to become a perfect square.

$1200 = 3 \times 2^2 \times 2^2 \times 5^2$

Here, 3 appears only once as the factor of 1200. Thus if we multiply 1200 by 3, then all the factors appear even number of times.

Hence the required least number is 3.

Example 4. Find the least number by which 9900 must be divided to make the quotient a perfect square.

$9900 = 2^2 \times 3^2 \times 5^2 \times 11$ (Verify yourself by resolving into factors)

Hence, 9900 must be divided by 11 to make the quotient a perfect square.

Example 5. Find the least square number which is divisible by 25, 40 and 60.

L.C.M. of 25, 40 and $60 = 600$

Now $600 = 2 \times 2 \times 2 \times 3 \times 5 \times 5$

$= 2^2 \times 2 \times 3 \times 5^2$

We know that any multiple of 600 is also divisible by 25, 40 and 60.

In order to make a multiple of 600 a perfect square, it should be multiplied by 2×3 i.e. 6.

Hence required number $= 600 \times 6 = 3600$

Example 6. Each student of a class subscribed as many 5 paise as the number of students were. The total subscription raised was Rs. 101·25 p. Find the number of students in the class.

Rs. 101·25 p. = 10125 paisa

10125 p. ÷ 5 p. = 2025

Rs. 101·25 is equivalent to 2025 number of 5 paise.

Now $\sqrt{2025} = 45$

∴ There were 45 students in the class.

Exercise—27

Find the square root by the method of factorization :

1. 576 2. 676 3. 784 4. 1024
5, 1225 6. 1600 7. 2025 8. 3600
9. 1089 10. 2304 11. 5625 12. 4356
13. 7056 14. 9801 15. 11025 16. 11664

17. Total contribution of a club towards flood relief was Rs. 1764. What was the total number of members of the club, if the contribution of each member was as many paise as the total number of members of the club ?

18. Find the least whole number by which 6300 is to be multiplied in order to make it a perfect square.

19. Find the least whole number by which 2744 is to be divided to make the quotient a perfect square.

20. Find two numbers whose product and qnotient are 12348 and 7.

21. The product of first and second number is 24, that of second and third number is 48, and that of first and third is 32. Find the numbers.

22. Find the least square number which is divisible by 12, 16, 20 and 24.

23. A gardener wants to plant 7065 trees in a garden in such a way that the number of trees planted in each row is equal to the number of rows. When the trees are planted in such manner, the gardener finds that 9 trees are in excess. Find the number of trees planted in a row.

CHAPTER 6

Unitary Method

Suppose one of your friends has purchased 5 exercise books for Rs. 5.75. If you take 2 exercise books from him, how much should you pay for these 2 exercise books ?

To solve this problem, first find out the price of one exercise book by dividing the total cost by 5. Then you multiply the price of one exercise book by 2 to find out the price of 2 exercise books. This procedure can be written conveniently as follows :

The cost of 5 exercise books = Rs. 5.75

" " " 1 " " = Rs. $\frac{5.75}{5}$

" " " 2 " " = Rs. $\frac{\overset{115}{\cancel{5.75}}\times 2}{\cancel{5}}$

= Rs. 2.30

Thus the method applied to find out the price of several articles from the given cost of some given articles of the same kind, or to find out the weight of several articles from the total weight of some given articles of the same kind is known as **Unitary Method.** The solution of a problem by unitary method has following characteristics.

(1) Firstly, a relation between two quantities will be given. In the above example, a relation between the number of exercise books and their price is given. Secondly, one of those will be given and we will have to find the other from the given relation. For example, the number of exercise books in the above example is given as 2. From the given relation i. e. the total cost of 5 exercise books, we have to find the cost of 2 exercise books.

(2) To solve the problem methodically, first you think of what is to be found out. Then write the given relation in a statement in such a way so as to place the quantity given in the relation in the left hand side and the quantity which is to be found out in the extreme right of the statement. For example, in the above example cost of 2 exercise books has to be found out. Hence the statement has been written in such a way that the cost lies in the extrme right.

(3) Since the cost of 1 exercise book is less than the cost of 5 exercise books, we have to divide the total cost of 5 exercise books by 5, the total number of exercise books to obtain the cost of 1 exercise book. This relation has been expressed as a fraction without making any final calculation at the second step of the above example.

(4) In the last step of the above example, the fraction obtained in the second step has been multiplied by 2, as the cost of 2 exercise books is greater than that of 1 exercise book.

Thus unitary method involves the combined application of multiplication and division. From the given problem it should be properly understood where the method of multiplication or division is to be applied.

Example 1. The cost of 20 litres or kerosene oil is Rs. 58. How much kerosene oil can be purchased for Rs. 232?

For Rs. 58 the amount of kerosene oil obtained is 20 litres.

,, ,, 1 ,, ,, ,, ,, ,, is $\frac{20}{58}$ litres

For Rs. 232 the amount of kerosene oil obtained is $\frac{20 \times \cancel{232}^{4}}{\cancel{58}}$

$= 80$ litres.

Hence 80 litres of kerosene oil can be obtained for Rs. 232.

Remark :

In the above example, the amount of keroseine oil is asked for. Hence the first line has been written in such a way so as to place the amount of the oil i.e. 20 litres in the extreme right.

Application of Unitary Method

A. Profit and Loss

Suppose a shop-keeper has purchased an exercise book for 110 p. and sells it for 125 p. Hence the cost price of the exercise book to the shop keeper is 110 p. and the selling price is 125 p. By selling the exercise book, the shopkeeper makes a profit of (125 p. - 110 p.) = 15 p.

Then you see,

Profit = Selling price—Cost price

Again 110 p. + 15 p. = 125 p.

Hence **Cost price + Profit = Selling price**

Sometimes articles are to be sold at a lower price than the cost price. In that case, the seller loses. For example, if the shop-keeper purchases an exercise book at 110 p. and sells it a 105 p., then the loss is 110 p. — 105 p. = 5 p.

Hence **Loss = Cost price— Selling price**

Again 105 p. + 5p = 110 p.

Hence Selling price+loss=Cost price.

Unitary method helps us to solve problems concerning profit and loss.

Example 1. 15 pens are purchased for Rs. 105 and sold for Rs. 127·50. What is the profit per pen ?

Total selling price = Rs. 127·50
Total cost price = Rs. 105 00
∴ Total profit = Rs. 22·50
The profit for 15 pens = Rs. 22·50
∴ The profit for 1 pen = Rs. $\frac{22\cdot50}{15}$
= Rs. 1·50

Hence the profit per pen is Rs. 1·50.

Example 2. Tea is purchased for Rs. 175 at the rate of Rs. 70 per kg. At what price per kg. the tea should be sold to earn a gross profit of Rs. 30?

Toral cost price = Rs. 175
Profit = Rs. 30

∴ Total selling price = Rs. 175 + Rs. 30 = Rs. 205
Rs. 70 = cost price of 1 kg. of tea

Re. 1 = ,, ,, ,, $\frac{1}{70}$,, ,, ,,

Rs. 175 = ,, ,, ,, $\frac{175}{70}$,, ,, ,,

= $\frac{\overset{5}{\cancel{175}}}{\underset{2}{\cancel{70}}}$,, ,, ,,

= $\frac{5}{2}$,, ,, ,, ,,

Hence $\frac{5}{2}$ kg. of tea was purchased.

Therefore, selling price of $\frac{5}{2}$ kg. of tea = Rs. 205

∴ ,, ,, ,, ,, 1 ,, ,, ,, = Rs. $\frac{205}{5}$

= Rs. $\frac{205}{5}\times\frac{2}{5}$Rs. 82

Hence tea has to be sold at Rs. 82 per kg.

Alternative method:

When the cost price is Rs. 175, selling price is Rs. 205

„ „ „ „ „ Re 1 „ „ Rs. $\frac{205}{175}$

„ „ „ „ „ Rs. 70 „ „ Rs. $\frac{205 \times 70}{175}$

= Rs. 82

[Remark: Since Rs. 70 in the cost price of 1 kg., therefore the corresponding selling price is the selling price for 1 kg.]

Example 3. 16 kg. of the tea at the rate of Rs. 62 per kg. is mixed with 14 kg. of tea at the rate of Rs. 64 per kg. At what rate this mixed tea should be sold to have a gross profit of Rs. 14?

The cost price of 16 kg. of tea at the rate of Rs. 62
= Rs. 62 x 16
= Rs. 992
The cost price of 14 kg. of tea at the rate of Rs. 64
= Rs. 64 x 14 = Rs. 896
Total cost price = Rs 992 + Rs. 896
= Rs. 1888
Profit = Rs. 14
Total selling price = Rs. 1888 + Rs. 14
= Rs. 1902.

Total amount of tea sol =(16+14) Kg. = 30 kg.

The selling price of 30 kg. of tea = Rs. 1902

„ „ „ „ 1 kg. of tea = Rs. $\frac{1902}{30}$ = Rs. 63.40

Hence tea is to be sold at the rate of Rs. 63.40 per kg.

Example 4. 240 lemons are bought at the rate of 6 per Rupee and another 240 lemons at the rate of 8 per Rupee.

All the lemons are sold at the rate of 5 per Rupee. What will be the profit or loss ?

In the first case,

Cost price of 6 lemons = Re. 1

" " " 1 lemon = Re. $\frac{1}{6}$

" " " 240 lemons = Rs. $\frac{240}{6}$

= Rs 40

In the second case,

Cost price of 8 lemons = Re. 1

" " " 1 lemon = Re. $\frac{1}{8}$

" " " 240 lemons = Rs. $\frac{240}{8}$ = Rs. 30

Total cost price = Rs. 40 + Rs. 30

= Rs. 70

Total number of lemons = 240 + 240 = 480

Now,

Selling price of 5 lemons = Re. 1

" " " 1 lemon = Re. $\frac{1}{5}$

" " " 480 lemons = Rs. $\frac{480}{5}$

= Rs. 96

∴ Profit = Rs. 96 − Rs. 70 = Rs. 26.

Exercise—28

1. The cost price of 10 pens is Rs. 6·15. What will be the selling price of each pen, so as to make a net profit of Rs. 10·50 ?

2. If 5 quintals of potato are purchased at the rate of

Rs. 190 per quintal, at what rate per kg. of potato should be sold to earn a net profit of Rs 100?

3. If a cow is sold at Rs. 1725, then the loss is Rs. 115. If 10 cows are sold at a profit of Rs. 500, what will be the selling price of each cow?

4. A man buys 10 dozens of eggs at the rate of Rs. 17.40 per dozen. He sells 6 dozens at the rate of Rs. 16.80 per dozen and the rest at Rs. 18 per dozen. What is his profit or loss? What would be his profit or loss by buying and selling 25 dozens of eggs in the above manner?

5. A man buys lemons for Rs. 50 at the rate of 3 lemons per Rupee and sells them at the rate of 5 for 2 Rupees. What is his profit?

6. 10 kg. of tea at the rate of Rs. 66 per kg. is mixed with 10 kg. of tea at the rate of Rs. 62 per kg. What should be the selling price of the mixed tea per kg., so that there will be neither profit nor loss?

7. A man sold 50 buffaloes at the rate of Rs. 2825 each and finds that he sustains a loss of Rs. 1750. What is the cost price of each buffalo?

8. A dealer purchases 80 quintals of oil at Rs. 1100 per quintal and sells 60 quintals at Rs. 1150.50. At what rate should be the remaining oil be sold, so as to make a net profit of Rs. 1000?

9. Equal amounts of tea of two kinds, one at Rs. 66 per kg. and the other at Rs. 68 per kg. are mixed. Find the amount of each kind of tea, if the mixed tea is sold at Rs. 67.25 to make a profit of Rs. 12.

10. A shopkeeper sustains a loss of $\frac{1}{5}$th of the cost price by selling a watch. If he would sell it at Rs. 70 more, he would have made a profit of $\frac{4}{15}$th of the cost price. What is the cost price of the watch?

B. Time and Work

From the discussion of unitary method in the last chapter, we know that the price or weight of one article can be found out by division, if the price or weight of few articles and the number of articles are known. Once we know the price or weight of one article, we can find the price of several articles of the same kind by multiplication. This process becomes somewhat different, if we want to solve the problems of time and work.

Suppose, 3 farmers can harvest a plot in 2 days. Now instead of 3 farmers, if 1 farmer would have been engaged for the same work, the time taken by the farmer would have been 3 times the time taken previously, as he alone would have to perform the work of 3 farmers. Hence the total time required would have been 3 times i.e. 2 × 3 or 6 days.

Therefore, you observe that when 3 men do a piece of work in 2 days, then to find the number of days in which one man can do it, 2 is to be multiplied by 3.

Again suppose, an artisan taken 10 days to paint the doors and windows of your school building. If 2 artisans would have been employed for this job, the amount of work would be shared by them. As such the time taken by 2 artisans to do the job would have been half the time taken by 2 artisans i.e. they would take $\frac{10}{2} = 5$ days. We assume that each man can do same work at same time i.e. their efficiency is same.

From the above discussion, it is clear that if we decrease the number of workers, the total time to finish the work will increase, if the amount of work remains the same. Hence in this case, the operation of multiplication is to be performed. On the other hand if we increase the number of workers, the time will decrease and the operation of division is to be performed in that case.

Example 1. 10 men can do a piece of work in 12 days. How many days will 8 men take to do the same work ?

10 men can do the work in 12 days
1 man ,, ,, ,, ,, ,, 12×10 days
8 men ,, ,, ,, ,, ,, $\frac{12 \times 10}{\cancel{8}}$ days
$= 15$ days

∴ 8 men will do the work in 15 days.

Example 2. 12 farmers take 5 days to harvest a plot of land. How many farmers can harvest the same land in 4 days ?

5 days are required to harvest a plot of land by 12 farmers
1 day is ,, ,, ,, ,, ,, ,, ,, ,, 12×5 ,,
4 days are ,, ,, ,, ,, ,, ,, ,, ,, $\frac{\overset{3}{\cancel{12}} \times 5}{\cancel{4}}$,,
$= 15$ farmers

∴ 15 farmers will harvest in 4 days.

Example 3. A does a work in 12 days and B in 8 days. In how many days will they together do the work ?

In 12 days A does the work
,, 1 day A does $\frac{1}{12}$th part of the work
In 8 days B does the work
,, 1 day B does $\frac{1}{8}$th part of the work

Hence, in 1 day A and B together do $\left(\frac{1}{12} + \frac{1}{8}\right) = \frac{5}{24}$th part of the work.

Now A+B do $\frac{5}{24}$th part of the work in 1 day.

∴ „ „ total work in $(1 \div \frac{5}{24})$ days

$= \frac{24}{5}$ days

$= 4\frac{4}{5}$ days

Hence A and B together do the work in $4\frac{4}{5}$ days.

Example 4. A can do a piece of work in 6 days and B can do the work in 9 days. They together started the work and after 2 days A left. How many days more will B require to finish the work?

In 6 days A can do the work

„ 1 day A can do $\frac{1}{6}$th part of the work;

In 9 days B can do the work

„ 1 day B can do $\frac{1}{9}$th part of the work;

In 1 day (A+B) together can do $(\frac{1}{6}+\frac{1}{9})=\frac{5}{18}$th of the work

In 2 days they can do $\frac{5}{18}\times 2=\frac{5}{9}$ th of the work;

Remaining part of the work $= 1-\frac{5}{9}=\frac{4}{9}$ part

This work has to be done by B alone.

B can do $\frac{1}{9}$th part of the work in 1 day

∴ „ „ „ whole work in 9 days

∴ „ „ $\frac{4}{9}$th part in $9\times\frac{4}{9}$ days

$= 4$ days;

Hence, after A left the work, B will finish the work in 4 days.

Exercise—29

1. 9 men can do a work in 21 days. How many days will 7 men take to do the same work?

2. How many labourers can finish a work in 45 days, if 12 labourers take 30 days to finish it?

3. 42 labourers can carry 5000 bricks in 30 days, How many days will be required to carry these bricks by 18 labourers ?

4. 20 people can dig a canal in 15 days. In how many days 30 people will dig the canal ? How many people will be required to dig it in 6 days ?

5. 30 labourers started constructing a road with a contract to finish it in 21 days. After in 10 days what fraction of the work remains unfinished ?

6. 250 labourers require 24 days to make a barrage. How many additional labourers are to be engaged to finish the barrage in 15 days ?

7. A can do a work in 12 days and B in 9 days. In how many days can they together do the work ?

8. A man working 8 hours daily take 12 days to finish a work. If the work is to be finished in 16 days, how many hours daily will he have to work ?

9. Nepal can do a work in 4 days and Gopal in 6 days. They finished the work in 2 days with the help of Ashok. In how many days can Ashok finish it alone ?

10. A, B and C can do a work in 10, 12 and 15 days respectively. They started together to do the work. A left 5 days before the work is finished. How many days will be required to finish the work ?

Objective and short answers type questions

1. State wheather the following statements are 'true' or 'false' ?

(a) All odd numbers are not prime.

(b) No even number is prime.

(c) All multiples of 6 are also multiples of 18.

(d) Both the multiplier and multiplicand are factors of their product.

(e) The product of a number and its reciprocal is greater than unity.

(f) The reciprocal of $\frac{13}{21}$ is $\frac{21}{13}$.

(g) The square root of the square root of 81 is 3.

(h) The more the selling price than the cost price is, the less is the profit.

2. There are four answers to each question. Choose the correct answer.

(a) L. C. M. of 48 and 60 is :

(i) 2×3 (ii) $2 \times 2 \times 3$

(iii) $2 \times 2 \times 2 \times 2 \times 3 \times 5$

(iv) None of these.

(b) Every integer will have at least the following set of factors :

(i) 0, 1 (ii) 0, the number itself

(iii) 1, the number itself (iv) None of these.

(c) The odd number among the following number is.

(i) 15×16 (ii) 18×13 (iii) 12×1

(iv) None of these.

(d) The number of ways of expressing 175 as the product of two whole numbers is :

(i) 2 (ii) 3 (iii) 4 (iv) 5

(e) The sum of three numbers is a two digit number. Among the numbers :

(i) at least two should be less than 50 (ii) at least one should be less than 50 (iii) all the numbers are equal (iv) None of them is greater than 50.

(f) Which of the following is greatest ?

(i) $\frac{1}{2}$ (ii) $\frac{2}{3}$ (iii) $\frac{3}{4}$ (iv) $\frac{5}{6}$

(g) Which of the following is least ?

(i) ·12 (ii) $\frac{1}{4}$ (iii) $\frac{1}{5}$ (iv) ·099

(h) Which of the following is equal to $\frac{1}{4}$?

(i) ·4 (ii) $\frac{1}{\frac{1}{4}}$ (iii) $\frac{1}{2} \div \frac{1}{2}$ (iv) None of these.

(i) From square root of which number, 4 is to be subtracted to make the difference 4.

(i) 4 (ii) 8 (iii) 16 (iv) 64.

(j) To complete a certain work by some persons

(i) Number of days required will be more if number of persons is increased.

(ii) Number of days required will be less if number of persons is decreased.

(iii) Number of days required will be less if number of persons is increased.

(iv) Number of days will be same as the number of persons.

(k) There is a collection of a sum of Rs. 64 from students, each paying the same number of rupees as the number of students. Then the number of students must be :

(i) 4 (ii) 8 (iii) 16 (iv) 32

3. Put a tic (√) mark against the correct statements.

(i) $7+5=5+7$

(ii) $7-5=5-7$

(iii) $7\times5=5\times7$

(iv) $7\div5=5\div7$

(v) L. C. M. of two numbers = H. C. F. of these two numbers.

(vi) The number, whose sum of the digits is divisible by 9, is also divisible by 9.

(vii) L. C. M. of two numbers × H. C. F. of those two numbers = Product of the same two numbers.

(viii) One of the factors of 35 is 7.

(ix) 5 is a multiple of 35.

(x) 2 is a prime number.

(xi) H C. F. means Highest Common Factor.

(xii) L. C. M.means Least Common Factor.

(xiii) Dividend = Divisor × Quotient − Remainder.

(xiv) $\text{Quotient}=\dfrac{\text{Dividend}-\text{Remainder}}{\text{Divisor.}}$

(xv) A fraction remains unaltered if the numerator and denominater are multiplied by the same number.

(xvi) A fraction remains unaltered if the same number is added to its numerator and denominator.

(xvii) $\frac{3}{4}\div\frac{1}{2}$ of $\frac{2}{3}=\frac{3}{4}\div\frac{1}{2}\times\frac{2}{3}$

(xviii) $\cdot0001\times\cdot0001=\cdot0000001$

(xix) $\cdot0001\div\cdot01=\cdot01$

(xx) $5\div\frac{1}{4}(\frac{1}{6}+\frac{1}{3})=5\div(\frac{1}{6}+\frac{1}{3})$ of $\frac{1}{4}$

(xxi) If the denominator of a fraction, expressed in its lowest term, has a factor 7, then the fraction if converted into a fraction will be a recurring decimal.

(xxii) Square of square root of 4 is 2.

(xxiii) The numbers having last two digits as 25, are perfect squares.

(xxiv) If 4 persons can make 4 mats in 4 days, then 12 persons can make 12 such mats in 12 days.

4. Answer the following :

(i) Find the number of such prime numbers which has 5 as the last digit.

(ii) When are two numbers said to be relatively prime ? Give examples.

(iii) How many perfect squares are there in between 2 and 50 ? Write down these square numbers.

(iv) What is the square root of square of 16 ?

(v) Find the number whose square, when added to square of 4, become square of 5.

(vi) $1 \div \frac{1}{2}$ of $2 \times \frac{1}{3} - \frac{1}{4} + \frac{1}{6} = ?$

(vii) $\frac{1}{2}$ of $2 \div \frac{1}{2} \times \frac{1}{2}(\frac{1}{2} + \frac{1}{2}) = ?$

(viii) $\frac{1}{2} + \frac{1}{2}[\frac{1}{2} + \{\frac{1}{2}(\frac{1}{2} + \frac{1}{2})\}] = ?$

(ix) $37\frac{1}{2} \div \frac{1}{4}$ of $3\frac{1}{3} = ?$

(x) $3\frac{1}{3}$ of $\frac{1}{4} \div 37\frac{1}{2} = ?$

(xi) Divisibility by which numbers is to be tested to know whether 191 is prime or not ? Why is it not necessary to test divisibility by other numbers besides these numbers ?

(xii) Find the common factors of 28 and 40. Which is the greatest common factor ?

(xiii) If we write the numbers from 1 to 100 consecutively, how many times we have to write the digit 'zero' ?

(xiv) Evaluate the following and find the greatest and the least :

(a) $33 \div 3 - 3$.

(b) $\frac{3}{3} + \frac{3}{3}$,

(c) $\frac{3}{3} + \sqrt{3 \times 3}$,

(d) $33 \div 3 + 3$,

(e) $(33 - 3) \div 3$.

(xv) Fill up the gaps in the following table by inserting appropriate numbers.

	Dividend	Divisor	Quotient	Remainder
Example	11	4	2	3
1	14	3		
2	39		5	4
3	56	6		
4	74		3	
5		9	13	2
6	98	9		
7		15	15	14
8	235		23	

(xvi) Fill in the following tables by inserting appropriate numbers from 1 to 20.

Number	Numbers in between 1 to 20
Even numbers	
Odd numbers	
Prime numbers	
Composit numbers	
Factors of 20	
Multiple of 20	

(xvii) 24 trees are to be planted in some rows. Each row will contain same number of trees. How many such arrangements are possible, considering that each row will contain more than one tree and less than 24 trees ?

(xviii) Express $\frac{130}{125}$ as a decimal fraction.

(xix) Express the sum of $\left(\frac{1}{10}+\frac{1}{100}\div\frac{1}{1000}\right)$ in decimal fraction.

(xx) The cost of 2 metres tape is 75 p. in one shop and that of 3 metres tape is Rs. 1·12 in another shop. In which shop the rate of the tape is cheaper ?

(xxi) A farmer produces $51\frac{5}{8}$ quintals of paddy in a year from one piece of land. If he requires $32\frac{1}{2}$ quintals of paddy for his family, find the surplus amount of paddy he has.

(xxii) The students of class VI of Binodini Girl's School are to be arranged in a square and 5 students were in excess. If the number of students is in between 40 and 50, what will be the number of students ?

(xxiii) Rice is purchased at the rate of Rs. 6.40 per kg. and is sold at the rate of Rs. 6.40 per kg. What will be the profit per quintal ?

(xxiv) A person bought 5 lemons for one rupee and sold at the rate of 4 lemons per rupee. If he has purchased lemons for Re. 1·00, what would be his profit ?

(xxv) The speed of first bus is 50 km. per hour and that of second bus is ·9 km. per minute. Which bus moves faster ?

(xxvi) Is eleven thousand one hundred and eleven divisible by 3 ?

(xxvii) The cost of a bottle with its cork is Rs. 1·10. The bottle costs one rupee more than the cork. What is the cost of the bottle ?

Miscellaneous Exercise

1. The weight of a glass full of water is 100 gms. The empty glass weighs 45 gms. 320 mgs. What will be the weight of water only, when the glass is full of water ?

2. The weight of a metal box when empty is 10 kg. 320 gms. The box weighs 23 kg. 40 gms. when 24 packets, each of same weight are placed inside the box. Find the weight of each packet.

3. From an iron sheet of one metre length, 17 pieces, each of length 4 cm 3 mms, are cut out. If during each cut, 2 mm length of metal is lost, find the length of the portion left.

4. The road between Banamalipur and Ramnagar is of length 30 kms. If there are four bridges each of length 475 metre on this road, what will be the length of the road without the bridges ?

5. If the cost of cementing a floor is Rs. 3·75 p. per square metre, what will be the cost for cementing a floor of 165 sq. m. area ?

6. Find the factors of 60.

7. Find whether 499 is prime or not.

8. Among how many children can 70 chocolate be distributed equally without breaking into pieces ? Give all possible answers.

9. Find the greatest and least numbers of 5 digits, which are divisible by 73.

10. The circumferences of front wheel and back wheel of a car are 144 cm and 162 cms respectively. Find the least distance covered by the car if both the wheels make complete revolution.

11. Some fishermen caught 455 trout fishes and 245 salmon fishes. They distributed both kinds of fishes equally among themselves. What may be the numbers of fishermen ?

12. A number was multiplied by 3 and the product was added to $\frac{3}{4}$th of it. The sum was divided by 7 and $\frac{1}{3}$rd of the quotient is subtracted from the quotient. The difference thus obtained was multiplied by this number. 52 was subtracted from the product and the square root of the remainder was calculated. 8 was added to the square root and the result was divided by 10 and the quotient was found to be 2. What was the original number ?

13. Simplify —

$1 \div \frac{1}{3}$ of $\frac{1}{2} \times \frac{1}{4} - \frac{1}{5} + \frac{1}{6}$.

14. A tram runs at the rate of $53\frac{1}{5}$ kms. per hour. How much time will it take to move 798 km. ?

15. $\frac{1}{5}$th of the potatoes produced by a farmer from his land was decomposed. He sold $\frac{1}{3}$ of the remaining potatoes. $\frac{5}{8}$th part of the remaining portion were consumed by his family and the rest amounts to 6 quintals. How much potatoe did the farmer produce from his land ?

16. The daily wages of two employees were Rs. 40 and Rs. 37 respectively After some days, it was found that the first employee had earned Rs. 100 more than the second employee. For how many days did they work ?

17. $\frac{1}{2}$ of the total books of a library are novels and $\frac{3}{4}$ of the remaining are on poetry. If the remaining 150 books are for adults, find the total number of books in the library.

18. In a village panchayat, $\frac{1}{2}$ of the total allotment was spent for repair of roads. Two third of the remaining was spent for agricultural loan. The remaining amount was given as grant to three co-operatives equally. If each cooperative gets Rs. 2000·00, find the total allotment to the panchayate.

19. A certain amount of money was distributed among Kamala and Ramala such that $\frac{4}{5}$th of the amount received by Kamala is equal to $\frac{7}{8}$th of the amount received by Ramala. If Kamala gets Rs. 525, what would be Ramala's share ?

20. $\frac{5}{8}$th of the total population of a village is adults and half of them is male. $\frac{4}{5}$th of the female adults as illiterate. Find the total population, if the number of illiterate female is 400.

21. A clerk has a monthly income of Rs. 1920 where as the monthly expenditure is $\frac{7}{8}$th of his income. There was an increase of $\frac{1}{6}$th of his present income but $\frac{2}{7}$ of the previous expenditure was increased due to rise in price. How much more or less will be his savings ?

22. The birth rate of a town is one tenth of the population where as $\frac{1}{20}$th of the population dies. If the population after two years is 44100, what was the original population ?

23. Rambabu clears $\frac{3}{20}$th of his debt every month. After clearing debt for 6 months, the debt remained unpaid was Rs. 290. What amount did he paid every month ?

24. Arrange in order of magnitude.
(descending and ascending)

$\frac{7}{25}, \frac{1}{3}, \frac{11}{30}$

25. (a) $\cdot 1 - \cdot 01 \times \cdot 0001$ of $\cdot 001 + \cdot 001 - \cdot 0001$

(b) Find the simplest value of

$$\frac{1 \cdot 69 \times 1 \cdot 69 - \cdot 31 \times \cdot 31}{1 \cdot 69 - \cdot 31}$$

26. If 5·5 litres of kerosene oil weighs 4·455 kg. what is the weight of 1 litre of kerosene oil ? How many litres of Kerosine oil weighs 4·05 kg. ?

27. The cost of 1 kg. of brinjal is Rs. 1·35. Find the cost of 5 kg. 600 gms. of brinjal.

28. The circumference of a wheel is 3·425 metre. What length will the wheel cover after making 16 complete revolutions ?

29. If 1 metre of cloth costs Rs. 5·20 p, what is the cost of 16 m 10 cm of cloth ?

30. The number of total voters in a polling booth is 1575 of which ·4 part is male. A candidate gets votes from ·6 part of male voters and ·4 part of female voters. Find the total number of votes the candidate gets.

31. In a garden the number of rows is same as the number of seedlings planted in each row. If there are 961 seedlings in total, find the number of seedlings planted in each row.

32. The boys of a school can be arranged in a square. The girls of the school also can be arranged in a square. The boys can also be arranged in 3, 4, 6, 8 rows and girls also in 2, 4, 5 and 10 rows. Find the least number of students in the school.

33. The members of a club collected subscription amounting to Rs 200; each paid as many 50 paise as the number of members in the club. Find the number of members in the club.

34. Each member of the executive committee of a library donated same number of rupees and same number of paise as the number of members in the executive committee. If the total collection was Rs. 122·21 p, find the total number of members of the executive committee.

35. If the square of the square of a number is the square root of square root of 65536, find the number.

36. 10 persons can repair 9 metres of a road in 5 days. 25 persons of the village performed the repair work for 10 days and received Rs. 2250 from the village Panchyate. Find how many km of road was repaired and what was the wages of each workers, if they divided the sum equally among themselves. Find also the cost of repairing 1 km length of road, assuming rate of wage is same.

37. Latif, Harish, Peter and Subal completed renovation work of a pond as a part of programme of test relief and received Rs. 3200. Accounts of their work are tabulated below:

Name	Number of days worked	Number of working hours per day	Total number of hours worked	Wages
Latif	30	6		
Harish	20	7		
Peter	25	8		
Suball	35	8		
Total				

Put the total number of working hours of each person in above table. What was the total working hours in all? What will be the rate of wages per hour? Insert the wages of each person in the appropriate column of the above table.

38. In a school boarding there are 50 students and rice was purchased for 40 days. After 10 days, 25 more students came to the boarding. How many days will be provided with the remaining rice?

39. Kamal can do a work in 10 hours whereas Amal can do the same in 12 hours. In what time will the work be completed when both work together?

40. The monthly wage and number of working days of 3 persons are tabulated below. Take 1 month = 30 days.

Name	Monthly wage	Number of working days	Wage
Sudhir	1350	39	
Suresh	1170	45	
Biman	1980	25	
Total			

Find the total wage of each and put them in respective places under the wage column in the above table. How much money will be required for their wages ?

41. 25 persons can do a job of making toys in 16 days. 10, persons left after working for 4 days. In how many days can the rest of the work be completed by remaining persons ?

42. 85 persons can build a house in 75 days. If the house is to be built in 51 days, how many more persons should have to be engaged ?

43. If 2 persons can make 2 chairs in 2 days, then how many chairs can be made by 60 persons in 60 days ?

44. A bus runs 125 km. in $2\frac{1}{2}$ hours. What time will it take to run 375 kms. ?

45. At present 3 oranges are available at Rs. 4. Before, 4 oranges were available at Rs. 3. If a person brought 10 oranges

at present rate, how much excess he had to pay compared to the previous rate ?

46. The cost of sugar in the market is Rs. 11.00 per kg. The cost of sugar in a fair price shop is Rs. 8.10 per kg. The monthly requirement of sugar for a family is 4 kg. 600 gms. If full quantity of sugar is purchased from market instead of purchasing from the fair price shop, what amount in excess will he have to pay ?

47. Same quality of cloth is sold in two shops. The selling price was fixed as Rs. 15.50 per metre and Rs 16.50 per metre at the first and second shop respectively. But the second shop takes $\frac{1}{30}$ th less at the time of sale. To buy 4m. 40 cms. of cloth, which shop will be profitable ? What will be the amount of profit ?

48. Assume that the price of commodities increases $\frac{1}{10}$th part every year. If the present price of an umbrella is Rs. 7.5 (i) What would be its cost after two years ? (ii) What was its cost 2 years ago ?

49. Assuming that monthly wage increases at the rate of $\frac{1}{15}$th part per year, the monthly income and expenditure of a person are Rs. 1650 and Rs. 1575 respectievly. There is an increase of expenditure in next year by $\frac{1}{10}$th part due to rise in price. What was his savings in first year and what will be his savings in next year ?

50. At most 9 gms. of sugar can be dissolved in 100 gms. of water. If 20 gms. of sugar is mixed with 140 gms of water, how much of sugar will remain undissolved ?

51. The height of Mount Everest is 29,000 ft. Express its height in kilometres and metres assuming 1 m = 3·28 ft.

52. The value of a dollar (American money) is Rs. 8·14 in Indian currency. What will be the value of 75 dollars Indian currency and will be the value of Rs. 407 in American currency ?

53. A map was drawn with the scale of 1 cm as 1·5 kms. If the distance between two places in the map is 4·3 cm, find the actual distance between two places.

54. Rahim, Gobinda and Ganesh sat together to take their lunch during a journey. Rahim had 4 breads, Gobinda had 3 breads, whereas Ganesh had no bread at all. They shared the breads equally among themselves. After taking their meal, Ganesh paid 70 paise in all to other two. Find how much money would each of Rahim and Gobinda receive?

55. 10 nails were fixed in an wooden plank 10 metres long, in such a way that 2 nails were fixed at the two ends and the other nails were fixed into the plank at equal distances from each other. If each nail requires a space of $\frac{1}{5}$ cm, find the distance between any two consecutive nails.

56. Pins were fixed in a plank at distances $3\frac{1}{4}$ cm a part. The distance between 1st and last pin was 3 m. $41\frac{1}{4}$ cm. How many pins were fixed? [Assume that no space was required by the pins.]

57. Some students collected $36\frac{7}{8}$ kg. of rice for flood relief. To make the total collection of 50 kg, how much rice will they have to collect more?

58. A table has four legs and $1\frac{1}{3}$ metres is to cut out from a piece of wood for each leg. If the length of the piece of wood is 8 metres, how much will be left when required amount of wood is cut out?

59. An iron rod of length 99·5 m. weighs 5·97 kg. What is the weight of such a rod of length 1 m.?

60. An empty pot weighs 65·32 gms. Some milk is poured into it and the weight is 99·24 gms. Now some sugar is mixed with that milk and the pot containing the milk mixed with sugar weighs 102·34 gms. Find the weights of milk and sugar separately.

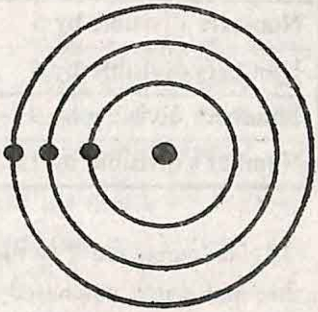
61. Amal, Kamal, Shyamal and Bimal had a sum of Rs. 300 on the whole. The amount of Kamal is twice that of Amal whereas the amount of Bimal is three times the sum of money Amal and Kamal have. The amount of Shyamal is 4 times the amount of the other three. Find the amount of money of each person.

62. A business man has to pay sale tax three times on selling price. First he paid the sale tax which is $\frac{1}{3}$rd of the price, then he paid $\frac{1}{4}$th of the remaining and in the third time he paid $\frac{1}{5}$th of the remaining. Find the total selling price of the business man, if the total amount of sale tax he paid was Rs. 24.

63. A man took $\frac{1}{6}$th of the total mangoes himself and gave $\frac{1}{5}$th of the remaining to his wife. He gave $\frac{1}{4}$th of the remaining mangoes to his eldest son and $\frac{1}{3}$rd of remaining to his middle son; and $\frac{1}{2}$of the mangoes which still remained he gave to his youngest son. 3 mangoes were left after this distribution. Find the total number of mangoes.

64. The price of 9 organges and 7 guavas is Rs. 10.70 and the price of 7 oranges and 9 guavas is Rs. 10.10. Find the price of 1 orange and 1 guava.

65. Three planets are revolving round a heavenly body. On a day, they are seen on the sky in a straight line on the same side of that heavenly body. The planets take 87 days, 232 days and 1218 days to make one complete revolution round that heavenly body. Find the least number of days after which they will be seen on the same position.



Some Additional Problems

1. Add the numbers in the table row-wise and columnwise. Also Find the row-wise totals and column-wise totals.

Add row-wise total ——————→

				Total
Add	$11\frac{1}{4}$	$2\frac{1}{2}$	$8\frac{3}{4}$	
Column wise ↓	5	$7\frac{1}{2}$	10	
	$6\frac{1}{4}$	$12\frac{1}{2}$	$3\frac{3}{4}$	
Total				

2. Write in appropriate places the numbers 111, 123, 783 1065 that are divisible by the numbers mentioned in the table.

Numbers divisible by 3	
Numbers divisible by 4	
Numbers divisible by 5	
Numbers divisible by 6	
Numbers divisible by 9	
Number s divisible by 15	

3. Nita purchased $2\frac{1}{4}$ metres of rope at the rate of 40 p. per metre and Kartik purchased $3\frac{2}{5}$metre of rope at the rate of 25 paise per metre. How much was spent by them for purchasing rope ?

4. A bottle can contain $1\frac{1}{2}$ litres of water. The bottle with water weighs 1531·32 gms, whereas the weight of empty bottle is 31·32 gms. Find the weight of 1 litre of water.

5. A box made of paper weighs 10·5 gms. Some marbles are kept in the box and the weight becomes 241·5 gms. If a marble weighs 21 gms. find the number of marbles in the box.

6. A person spends $\frac{1}{5}$th of his income for rent of the house, $\frac{1}{3}$rd part for food and $\frac{1}{8}$th part for purchase of clothes and sets aside $\frac{1}{4}$th part of his income for miscellaneous expenses. He saves the remaining of his income. What is his savings per rupee ? What will be his savings per Rs. 100 ? If his monthly income is Rs. 1250·00, what would be his annual savings ?

7. In a running competition, Rama reached her destination in $13\frac{1}{4}$; sec. Uma reached the destination $2\frac{1}{8}$ sec. after Rama ; Suchandra reached the destination $1\frac{1}{2}$ sec. earlier than Uma. Find the time taken by Suchandra to reach her destination.

8. Harish Babu went to market at 7·30 A. M. He took $\frac{1}{2}$ hour for marketing, $\frac{5}{8}$ hour for bathing and taking meal and $1\frac{1}{4}$ hour for reaching office, His office starts at 10·30 A. M. How much earlier the reaches office before the office starts ?

9. A cycle was sold for Rs. 825 and the profit was same as the loss if it was sold for Rs. 638. Find the cost price of the cycle.

10. A person can send a message to 3 persons in 15 mins. Each of these three persons sends the same message to 3 different persons in 15 mins. If the message spreads in this

manner how many persons can know the message in 2 hours ?

11. Four spheres made of lead weigh respectively 5·23 gm. 4·28 gm. 6·38 gm., and 5·55 gm. 10 spheres of same weight were made by melting those four. Find the weight of each sphere thus made.

ANSWERS

Exercise—1

1. (a) 5000 metre (b) 2010 metre (c) 31 metre (d) 130 metre (e) 1070 metre.

2. (a) 2 km. (b) 7·256 km. (c) ·019 km. (a) 2·34 km.

3. (a) 2000 gm. (b) 530 gm. (c) 1050 gm. (d) 5·002 gm. (e) ·301 gm. (f) 100000 gm.

4. (a) 5·2 litre (b) 20·5 litre. (c) 5001 litre (d) ·005 litre (e) 54 litre.

5. (a) ·632 kg. (b) 150 kg. (e) ·53 kg. (d) 5000 kg. (e) 110·5 kg.

6. (a) 6 dm. 2 cm. 5 mm. (b) 5 kg. 2 Hg. 4 Dg. (c) 3 Dm. 2m. 4 dm. (d) 1 l. 2 dl. 2 cl. (e) 1 Dm. 5 m. 2 dm. 6 cm. 8 mm. (f) 1 gm.

7. (a) 4 km. 1 Hm. 6 Dm. 3 m. (b) 13 m. 9 dm. 5 cm. (c) 2 Quintal 11 kg. 400 gm.

8. (a) 5 kg. 990 gm. (b) 11 m. 90 cm. (c) 1 l. 990 ml.

9. (a) 30 km. 930 m. (b) 308 kg 750 gm. (c) 144 l.

10. (a) 5 kg. 500 gm. (b) 600 metre. (c) 12 l. 200 ml.

11. 10·18 m. 12. 4 l 13. 125 gm. of rice and 75 gm. of pulse

14. Rs. 86.50 15. 2600 men 16. 2 l. 800 ml. 17. 4 l.

Exercise—2

(1) 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20

(2) 31, 37, 41, 43, 47, 53, 59

(3) 37, 43, 67, 71, 127

(4) 1, 2, 4, 8, 16
(5) 1, 3, 5, 15, 25, 75
(6) 2, 2, 2, 2, 3, 3
(7) 2, 2, 3, 3, 17
(8) 3, 5 (9) 5, 13
10. (a) $2 \times 5 \times 5 \times 5 \times 5$ (b) $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$
(c) $2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 7$ (d) $2 \times 3 \times 3 \times 5 \times 5 \times 11$
(e) $3 \times 3 \times 3 \times 3 \times 5 \times 5 \times 5$
(11) 11, 17 (12) 65, 143 (13) Prime
(14) 1, 5, 7, 13, 35, 65, 91, 455.

Exercise—3

(1) 16 (2) 40 (3) 14 (4) 11 (5) 32 (6) 34 (7) 2 (8) 25
(9) 6 (10) 30 paisa (11) 220 gm. (12) 6 dm. 5 cm.
(13) 26 ml. (14) 240 (15) 4950 (16) 5460
(17) 18270 (18) 660 (19) 17850 (20) 92378 (21) 3990
(22) Rs. 57·75 (23) 497 m. 25 cm. (24) 3 metric ton.

Exercise—4

(1) 3 parts of 5 equal parts of bamboo, 9 metre, 6 metre.
(2) Rs. 3,60 p. (3) Rs. 20.00 (4) Girl, $\frac{1}{35}$ l.
(5) $\frac{10}{15}, \frac{9}{15}, \frac{1}{15}$ (6) $\frac{14}{96}, \frac{20}{96}, \frac{64}{96}, \frac{9}{96}$ (7) $\frac{60}{132}, \frac{77}{132}, \frac{32}{132}$
(8) $\frac{105}{175}, \frac{56}{175}, \frac{45}{175}, \frac{150}{175}$ (9) $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{6}{7}$ (10) $\frac{3}{10}, \frac{2}{5}, \frac{3}{7}, \frac{8}{9}$
(11) $\frac{3}{8}, \frac{5}{12}, \frac{5}{9}, \frac{4}{7}$ (12) $\frac{2}{7}, \frac{1}{3}, \frac{3}{5}, \frac{9}{10}$ (13) $4\frac{1}{6}$ (14) $\frac{5}{6}$
(15) $19\frac{1}{5}$.

Exercise—5

1. (a) ·4 (b) ·625 (c) 3·6 (d) 8·2 (e) 2·125 (f) 4·75
2. (a) $\frac{1}{100}$ (b) $\frac{1}{8}$ (c) $2\frac{2}{5}$ (d) $\frac{1}{500}$ (e) $14\frac{17}{20}$ (f) $2\frac{1}{100}$
(g) $\frac{143}{1000000}$.

3. (a) 17·363 (b) 140·941 (c) 131·45 (d) 279·9311

4. (a) 124·958 (b) ·6621 (c) 14·418

5. (a) 67 (b) 30·2 (c) 31·2 (d) 67·648 (e) 5.434 (f) ·2875 (g) 82·11

6. (a) ·36 (b) ·35 (c) ·001012 (d) 4·06 (e) ·047 (f) 2·961 (g) ·135

7. (a) 3 kg. (b) Rs. 79·20 (c) 6·25 litre (d) 135 (e) ·000005

8. ·05 9. 0·1 10. 50 men 11. 8 litre 12. 500 men

Exercise—6

1. (i) 37 paisa (ii) Rs. 1.85 (iii) Rs. 1.11 2.5
3. (i) 27.5 km. (ii) 66 km.
4. (i) Rs. 72.00 (ii) Rs. 180.00 (iii) Rs. 1800.00
5. 11250 sqm. 6. In 12 days 7 Rs. 1542.50
8. 40 days 9. Rs. 5580 10. Price of a cow Rs. 1725 price of a buffalo Rs. 2587.50 11. 84 paisa
12. 9843.75 litre.

Exercise—7

1. (a) 183, 546, 5070 (b) 224, 608, 1028, 5400 (c) 375, 935, 1600, 4605 (d) 372, 1074, 5274 (e) 119, 343, 2401 (f) 136, 536, 2064, 7000 (g) 126, 432, 1737, 8010 (h) 242 341, 2618, 60907 (i) 672, 1620, 20736 (j) 225, 1080, 1125 3645 (k) 702, 1620. 1764, 3060 (l) 940, 5070, 5400, 1600, 340, 7000, 8010, 1620, 3690 1080 2590 3060

2, (a) 2, 3, 4, 6, 11 (b) 3, 5 (c) 3, 9, (d) 3, 5, 11 (e) 3, 11 (f) 2, 4, 8 (g) 3, 9, 11 (h) 2, 3, 5, 6, 10 (i) 11 (j) 2, 3, 5, 6, 10

Exercise—8

1 (a) 11, 13, 17. 19, 23, 29. 31, 37, 41, 43, 47 (b) 101, 103, 107, 109. 113

2. (a) 2, 2, 3, 3, 3 (b) 2, 2, 3, 3, 7 (c) 3, 7, 13 (d) 7, 7, 7 (e) 2, 2, 2, 3, 3, 7 (f) 2, 3, 13, 17 (g) 2, 2, 3, 5. 5, 11 (h) 2, 2, 3, 3, 3, 5, 11 (i) 2, 2, 2, 3, 3, 5, 5, 11

3. (a) $2^2 \times 3 \times 11$ (b) $3^2 \times 5^2$ (c) $2 \times 3^3 \times 13$ (d) $2 \times 3 \times 5^2 \times 7$ (e) $2 \times 3^2 \times 5^2 \times 7$ (f) $2^2 \times 5 \times 37$ (g) $2^4 \times 3 \times 5 \times 7 \times 13$

4. (a) Prime (b) Prime (c) Not prime (d) Not prime

5, (a) Yes (b) Yes (c) Yes (d) No (e) Yes

6. (a) 12 (b) 450 (c) 504 (d) 10800 (e) 16200.

Exercise—9

1. (i) 5 (ii) 10 (iii) 15 (iv) 20 (v) 10 (vi) 5 (vii) 10 (viii) 20 (ix) 15 (x) 22 (xi) 20 (xii) 40 (xiii) 28 (xiv) 13 (xv) 28 (xvi) 17

2. (i) 37 (ii) 51 (iii) 12 (iv) 5 (v) 9 (vi) 121 (vii) 133 (viii) 151 (ix) 19 (x) 61 (xi) 6 (xii) 83

3. (i) 20 paisa (ii) 7 paisa (iii) 133 gm. (iv) 466 gm. (v) 25 cm. (vi) 99 cm. (vii) 10 kg. (viii) 229 ml. (ix) 7 ml.

4· 28 litre 5. 18 6. 40 cm.

Exercise—10

(1) 48 (2) 72 (3) 30 (4) 24 (5) 60 (6) 100 (7) 144
(8) 252 (9) 480 (10) 1764 (11) 3185 (12) 360
(13) 336 (14) 17160 (15) 36504 (16) 150 (17) 168
(18) 600 (19) 840 (20) 240 (21) 864 (22) 120
(23) 144 (24) 360 (25) 720 (26) 300 (27) 432

(28) Rs. 48.00 (29) Rs. 35.28 (30) 2 Quintal 54 Kg. 8 Hg. (31) 184 m. 68 cm. (32) hours 18 min. (33) 18, 720. equal (34) 28, 560, equal (35) 5, 450, not equal (36) 5, 600, not equal.

Exercise—11

(1) 300 (2) 5 (3) 33 (4) 4714, 234 (5) 24 (6) Re. 1·20 5, Sukla—2, Sukla's elder brother—3 (7) 91 (8) 3 A. M., 20 times, 15 times. 12 times. 10 times (9) 96 cm., 7. 10 (10) 540 families, Dhoti—2, Saree—3, Shirt—5 (11) 352 (12) 9980 (13) 360, 480 (14) 3595 (15) 84 (16) 25 (17) 671, 56, 61 (18) 336, 48 or, 240, 144 (19) At 11 A.M. Paikpara going bus—30 times Bagbazar going bus—20 times, Howarh going bus—15 times, Barrackpore going bus—12 times (20) 5001 (21) 40.

Exercise—12

(1) 50 p., 25 p., 20 p. (2) 50 p., Rs. 2, 60 p. (3) 2 days (4) 80 cm., 50 cm. (5) 25 min., 9 min, 8 min. (6) 4 kg.. 2 kg., 800 gm. (7) 2250 mi, (8) 120 p. (9) 3, 20, 12 (10) 8. 27, 30 (11) 84, 56, 90 (12) 52, 84, 108 (13) $\frac{16}{5}$ (14) $\frac{288}{15}$ (15) $\frac{573}{8}$ (16) $\frac{2212}{11}$ (17) $2\frac{1}{8}$ (18) $4\frac{3}{13}$ (19) $5\frac{10}{33}$ (20) $2\frac{7}{113}$ (21) $13\frac{1}{7}$ (22) $\frac{3}{5}$ (23) $\frac{13}{60}$ (24) $2\frac{5}{8}$ (25) $\frac{1}{4}$ (26) $\frac{3}{7}$ (27) $\frac{3}{35}$ (28) $\frac{16}{45}$ (29) $\frac{18}{30}, \frac{3}{30}, \frac{1}{30}$ (30) $\frac{6}{24}, \frac{20}{24}. \frac{14}{24}, \frac{1}{24}$ (31) $\frac{210}{735}, \frac{455}{735}, \frac{84}{735}, \frac{225}{735}$ (32) $\frac{8}{192}, \frac{16}{192}, \frac{30}{192}. \frac{21}{192}$ (33) $\frac{3}{10}, \frac{21}{30}, \frac{1}{3}, \frac{3}{5}$; $\frac{3}{5}, \frac{1}{3}, \frac{21}{30}, \frac{3}{10}$ (34) $\frac{5}{12}, \frac{3}{4}, \frac{6}{7}, \frac{7}{8}$; $\frac{7}{8}. \frac{6}{7}, \frac{3}{4}, \frac{5}{12}$. (35) $\frac{17}{24}, \frac{31}{36}. \frac{53}{60}, \frac{65}{72}$; $\frac{65}{72}, \frac{53}{60}, \frac{31}{36}, \frac{17}{24}$ (36) $\frac{12}{17}, \frac{43}{51}, \frac{29}{34}, \frac{94}{102}$; $\frac{94}{102}, \frac{29}{34}, \frac{43}{51}, \frac{12}{17}$ (37) $\frac{7}{12}, \frac{5}{16}$ (38) $\frac{5}{8}, \frac{3}{32}$ (39) $\frac{6}{7}. \frac{16}{21}$ (40) $\frac{25}{36}, \frac{125}{144}$.

Excrcise—13

(1) $1\frac{1}{12}$ (2) $\frac{71}{75}$ (3) $3\frac{5}{14}$ (4) $2\frac{79}{192}$. (5) $1\frac{71}{480}$ (6) $5\frac{71}{78}$ (7) $30\frac{41}{182}$ (8) $4\frac{41}{88}$ (9) $\frac{1879}{2520}$ (10) $8\frac{696}{3025}$ (11) $446\frac{9}{14}$ (12) 1000 (13) $8\frac{1}{30}$ (14) $16\frac{19}{48}$ (15) $1039\frac{5}{24}$ (16) $2\frac{1}{9}$ (17) $\frac{5}{6}$ (18) $\frac{75}{77}$ (19) $\frac{198}{199}$ (20) $\frac{7}{32}$ (21) $3\frac{1}{150}$ (22) $10\frac{5}{12}$ (23) $89\frac{1}{32}$ (24) $487\frac{1}{2}$ (25) $5\frac{1}{200}$ (26) $\frac{5}{6}$ (27) $1\frac{1}{3}$ (28) $\frac{1}{8}$ (29) $3\frac{7}{30}$ (30) $1\frac{1}{255}$ (31) $2\frac{57}{175}$.

Exercise—14

(1) $2\frac{1}{4}$, $3\frac{3}{4}$. 9, $13\frac{1}{2}$ (2) 0, $\frac{11}{12}$, $3\frac{2}{3}$, $5\frac{1}{2}$, $7\frac{1}{3}$ (3) $20\frac{1}{2}$, $61\frac{1}{2}$, 82, $102\frac{1}{2}$ (4) 4 men (5) 5000 (6) 1509 gm. (7) $1\frac{133}{600}$ km. (8) Rs. 175·21

Exercise —15

(1) $\frac{3}{5}$. $\frac{3}{25}$, $\frac{1}{10}$. $\frac{1}{15}$ (2) $1\frac{1}{14}$. $\frac{5}{7}$. $\frac{3}{7}$. $\frac{1}{7}$ (3) $2\frac{1}{5}$, $\frac{3}{5}$, $\frac{1}{5}$ (4) $\frac{7}{36}$ parts (5) The length of the least part of the rope is greater.

Exercise—16

(1) $\frac{3}{10}$ (2) $\frac{2}{7}$ (3) 3 (4) $\frac{\cdot 1}{12}$ (5) $\frac{1}{3}$ (6) $5\frac{2}{3}$ (7) 135 (8) 207 (9) 3876 (10) 5 (11) $\frac{\cdot 1}{30}$ (12) 20 (13) $12\frac{1}{2}$ (14) $1\frac{1}{4}$ (15) $\frac{5}{9}$ (16) $12\frac{1}{2}$ (17) $8\frac{3}{4}$ (18) $5\frac{1}{3}$ (19) 10 (20) 40 (21) $\frac{29}{100}$ (22) $2\frac{2}{3}$ (23) $\frac{44}{75}$ (24) $2877\frac{21}{187}$ (25) $31\frac{21}{25}$ (26) 20 (27) $\frac{7}{50}$ (28) 1 (29) $6\frac{402}{1445}$ (30) 3 (31) 1 (32) $\frac{1}{5}$ (33) $46\frac{7}{8}$ (34) $\frac{7}{30}$.

Exercise—17

(1) $3\frac{9}{16}$ (2) $3\frac{1}{2}$ (3) $\frac{1}{24}$ (4) $\frac{5}{18}$ (5) 4 (6) 219 (7) 29
(8) 0 (9) $5\frac{2}{5}$ (10) $\frac{7}{19}$ (11) $\frac{19}{480}$ (12) $1\frac{1}{2}$ (13) 1 (14) 0
(15) 1 (16) 2 (17) $2\frac{13}{24}$ (18) 1.

Exercise—18

(1) 6 (2) $\frac{1}{16}$ (3) 1 (4) $\frac{3}{4}$ (5) 7 (6) $\frac{5}{7}$ (7)1
(8) $2\frac{45}{152}$ (9) $1\frac{1}{4}$ (10) $\frac{4}{5}$ (11) 1 (12) $\frac{1}{5}$ (13) 2
(14) $\frac{5}{14}$ (15) $\frac{15}{52}$ (16) $\frac{21}{89}$ (17) $3\frac{d}{8}$ (18) $1\frac{1}{3}$ (19) 0·

Exercise—19

1. (a) $\frac{3}{5}$ (b) $\frac{1}{6}$ (c) $\frac{1}{4}$ (d) $\frac{1}{5}$ (e) $\frac{4}{23}$
2. (a) Rs 10·25 (b) 2 kg. 800 gm. (c) Rs. 44·24
(3) Rs. 30·00 (4) Rs. 900·00 (5) $\frac{3}{8}$ parts (6) $\frac{1}{15}$ parts.
Rs. 200·00 (7) 200 (8) Rs. 1200.00 (9) 24 km.
(10) 3000 (11) Rs. 400·00 (12) 192 kilolitre.
(13) 9 Quintal, 26 Quintal (14) Rs. 7000·00 (15) $1\frac{1}{2}$ kg.
(16) $\frac{1}{6}$ parts, $\frac{5}{6}$ parts (17) $\frac{8}{15}$ parts, $\frac{7}{15}$ parts (18) 150 hours
(19) Kamal gets 40 paisa, Rahim gets 10 paisa (20) Rs. 75.00
(21) $\frac{81}{140}$ parts, Asit=Rs. 150·00, Nihar—Rs. 122·50, Arati—
Rs. 77·50 (22) (a) $11\frac{4}{7}$ (b) 1.

Exercise—20

(1) 133·56 (2) 1498·616 (3) 788·162 (4) 3578·381
(5) 3·033 (6) 4·183 (7) 79·531 (8) 125·616 (9) 29·19

(10) 82·827 (11) 2·491 (12) 1·923 (13) 119·101
(14) 321·043 (15) 0·7562 (16) 866·013 (17) 86·632
(18) 352·711 (19) 2·057 (20) 3·417 (21) 50·407
(22) 3·504.

Exercise—21

1. (a) 42 (b) 32·8 (c) 70·15 (d) 0·03
2. (a) 10·2 (b) 23·5 (c) 89·23 (d) 9·52
3. (a) 28·5 (b) 102 (c) 640 (d) 80
4. (a) ·4 (b) 100·1 (c) ·08 (d) -24
5. (a) 1020 (b) 800 (c) 1210 (d) -3400
6. (a) 3100 (b) 1820 (c) 21 (d) 550
7. (a) 4·3 (b) ·31 (c) ·18 (d) ·423
8. (a) ·141 (b) ·3852 (c) ·0005 (d) ·11
9. (a) 3·43 (b) ·0127 (c) ·0003 (d) ·0108
10. (a) ·0005 (b) ·0103 (c) ·0008 (d) ·00004
11. (a) 1·825 (b) ·6182 (c) ·0011 (d) ·0005
12. (a) ·00005 (b) ·0412 (e) ·00001 (d) ·001008.

Exercise—22

(1) 93·75 (2) 8·268 (3) 0·2484 (4) 116·314 (5) 185·73
(6) 134·068 (7) ·61557 (8) 3·98099 (9) 395·2304
(10) 15·1823 (11) 29·7625 (12) 12·222 (13) 2·38896
(14) 287·069 (15) ·089444 (16) ·21189 (17) 54·36048
(18) 2·48688 (19) ·001188 (20) ·6545 (21) ·65016.

Exercise—23

(1) 40·5 (2) 3·02 (3) 32·15 (4) 11·34 (5) ·0185
(6) ·45 (7) 1·05 (8) ·116 (9) 1·28 (10) 1·05

(11) 204000 (12) 560 (13) ·084 (14) ·0245 (15) 322·5
(16) ·044 (17) 102·4 (18) 18·12 (19) 18·64 (20) 8·576.

Exercise—24

1. (a) 12·5 (b) 2·15 (c) ·375 (d) ·015 (e) ·003
(f) ·128 (g) 3·5625 (h) 4·0025 (i) ·8384 (j) 1·024

2. (a) $.\dot{6}$ (b) $.\dot{8}\dot{3}$ (c) 3.27 (d) 4.18 (e) $.42857\dot{1}$
(f) $.\dot{7}148\dot{5}$ (g) $3.\dot{5}$ (h) $.0\dot{5}\dot{4}$ (i) $.2\dot{3}$

3. (a) terminating (b) recurring (c) recurring
(d) terminating (e) recurring (f) recurring (g) terminating
(h) recurring . (i) recurring (j) recurring (k) terminating
(l) recurring

4. Pure recurring decimal : $\frac{7}{9}$, $3\frac{8}{27}$, $\frac{12}{33}$, $8\frac{9}{13}$
Mixed recurring decimal : $\frac{5}{24}$, $\frac{17}{65}$, $10\frac{10}{12}$, $\frac{7}{22}$.

Exercise—25

(1) ·2048 (2) 19·3375 (3) ·0003 (4) 1 (5) 3·1
(6) 21·9 (7) 1 (8) ·03 (9) ·8 (10) ·04 (11) 52·5
(12) 11·8 (13) ·1277 (14) 23·375.

Exercise—26

(1) Rs. 13.80,20 (2) Rs. 63.90, Rs. 6.10 (3) .04
(4) Rs. 3·50 (5) 8 hrs. (6) 1·92 kg. (7) 11·557 kg.
(11) Rs. 1500·00 (12) 551·6 mitre less (13) Rs. 360·00
(14) ·125 part increase.

Exercise—27

(1) 24 (2) 26 (3) 28 (4) 32 (5) 35 (6) 40 (7) 45
(8) 60 (9) 33 (10) 48 (11) 75 (12) 66 (13) 84
(14) 99 (15) 105 (16) 108 (17) 420 men (18) 7 (19) 14
(20) 294, 42 (21) First number—4, Second number—6,
Third number—8 (22) 3600 (23) 84.

Exercise—28

(1) Rs. 7.20 (2) 2.10 (3) Rs. 1890.00 (4) Rs. 1.20 loss
Rs. 3.00 loss (5) Rs. 10.00 (6) Rs. 64 per kg.
(7) Rs. 2860.00 (8) Rs. 998.50 (9) Each kind—24 kg.
(10) Rs. 150·00.

Exercise—29

(1) 27 days (2) 8 men (3) 70 days (4) In . 10 days, 50 men
(5) $\frac{11}{21}$ parts (6) 150 men (7) In $5\frac{1}{7}$ days (8) 6 hours
(9) In 12 days (10) In 6 days.

Objective and short answers type questions

1. (a) True (b) False (c) False (d) True (e) False
(f) True (g) True (h) False
2. (a) (iii) (b) (iii) (c) (iv) (d) (ii) (e) (i) (f) (iv)
(g) (iv) (h) (iv) (i) (iv) (j) (iii (k) (ii)
3. (i), (iii), (vi), (vii), (viii), (x), (xi), (xiv), (xv), (xix), (xx),
(xxi)
4. (i) one (ii) when the two numbers will have no common
factor except 1 ; 15, 16 (iii) 6 in number ; 4, 9, 16, 25, 36, 49
(iv) 16 (v) 3 (vi) $\frac{17}{60}$ (vii) 1 (viii) 1 (ix) 45 (x) $\frac{1}{45}$

(xi) 2, 3, 5, 7, 11 13, 17 ; for, when it is divided by the prime number 17, the quotient becomes less than 17
(xii) 1, 2, 4 ; 4 (xiii) 11 (xiv) 8, 2, 4, 14, 10 ; greatest number = 33 ÷ 3 + 3 ; least number = $\frac{3}{3}+\frac{3}{3}$ (xv) (1) Quotient 4, remainder 2 (2) Divisor 7 (3) Quotient 9, remainder 2 (4) Divisor 2, remainder 0 (5) Dividend 119 (6) Quotient 10, remainder 8 (7) Dividend 239 (8) Divisor 10. remainder 5
(xvi) 2, 4, 6, 8, 10, 12, 14, 16, 18, 20 ; 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, ; 2, 3, 5, 7, 11, 13, 17, 19 ; 4, 6, 8, 9. 10, 12, 14, 15, 16, 18, 20 ; 1, 2, 4, 5, 10, 20 ; 2, 4, 6, 8, 10, 12, 14, 16, 18, 20 (xvii) 6 (xviii) 1·04 (xix) 111 (xx) Second shop
(xxi) $19\frac{1}{8}$ quintal (xxii) 41 (xxiii) Rs. 20·00 (xxiv) 25 paisa
(xxv) Second bus (xxvi) No (xxvii) Rs. 1·05.

Miscellaneous Exercise

(1) 54 gm. 680 mgm. (2) 530 gm. (3) 235 mm. (4) 28 km. 100 m. (5) Rs. 618·75 (6) 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60 (7) Prime (8) 1, 2, 5, 7, 10, 14, 35, 70
(9) 10001, 99937 (10) 5 m. 76 cm. (11) 5, 7, 35 (12) 28
(13) $1\frac{7}{15}$ (14) 15 hrs. (15) 90 Quintals (16) 30 days
(17) 1200 (18) Rs. 36000·00 (19) Rs. 480·00 (20) 1600
(21) Rs. 160.00 will be less (22) 40000 (23) Rs. 435.00
(24) $\frac{11}{30}$, $\frac{1}{3}$, $\frac{7}{15}$; $\frac{7}{15}$, $\frac{1}{3}$, $\frac{11}{30}$ (25) (a) ·000901 (b) 2
(26) ·81 kg, 5 litres (27) Rs. 7·56 (28) 54·8 m. (29) Rs. 85·33 (30) 756 (31) 31 (32) 244 (33) 20 (34) 11
(35) 2 (36) 450 m. ; Rs. 90·00, Rs. 5000 00 (37) 180 hrs., 140 hrs., 200 hrs., 280 hrs. 800 hrs. Rs. 720.00, Rs. 560.00 Rs. 800.00. Rs. 1120.00, Rs. 3200.00 (38) 20 days (39) $5\frac{5}{11}$ hrs.
(40) Rs. 1755.00, Rs. 1755.00, Rs. 1650.00, Rs. 5160.00,
(41) In 20 days (42) 40 men (43) 1800 (44) $7\frac{1}{2}$' hrs.
((45) Rs. 7.00

(46) Rs. 13.34 (47) First shop, Rs. 1.98 (48) Rs. 90.75 Rs. $61\frac{119}{121}$ (49) Rs. 900.00, Rs. 300.00 (50) 7.4 gm. (51) 8 km. $841\frac{10}{41}$ m. (52) Rs. 610·50, 50 Dollar (53) 64·5 km· (54) Rahim will get 25 paisa, Govinda will get 10 paisa (55) 1 m. $10\frac{20}{37}$ cm. (56) 106 (57) $13\frac{1}{3}$ kg. (58) $5\frac{1}{3}$m., $2\frac{2}{3}$ m. (59) 6 kg. (60) 33·92 gm.. 3·10 gm. (61) Rs. 5·00. Rs. 10·00. Rs. 45·00, Rs. 240 00 (62) Rs. 40·00 (63) 36 (64) 80 paisa, 50 paisa (65) 4872 days.

Some Additional Problems

(1) Row-wise sum : $22\frac{1}{3}$, $22\frac{1}{3}$, $22\frac{1}{3}$ Column-wise sum : $22\frac{1}{3}$ $22\frac{1}{3}$, $22\frac{1}{3}$ Sum of all row-wise sums : $67\frac{1}{3}$ Sum of all .column-wise sums : $67\frac{1}{3}$ (2) 111, 123, 783, 1065 ; none of these ; 1065, none of these ; 783 ; 1065 (3) Re. 1·75 paisa (4) 1 kg. (5) 11 (6) 5 paisa per Rupee, Rs. 5·00, Rs. 750·00 (7) $14\frac{1}{12}$ sec. (8) 35 min (9) Rs. 731·50 paisa (10) 9841 men (11) 2·14 gm.

GEOMETRY

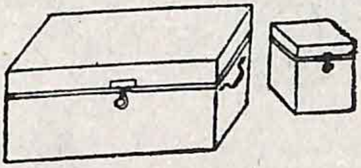
FIRST CHAPTER

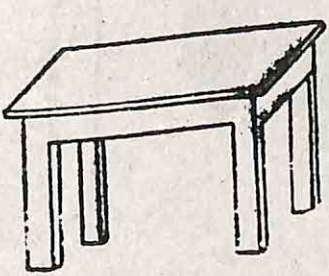
1. Solid Objects

Everyday you may have seen the following objects at home or at road or in the class room :

House, tree, a piece of coal, box, chair, table, book, exercise book, Almirah, unbroken brick, dice, drum, circular case ball, globe, tent, crickte bat, thatch, plantain flower, wax-candle, etc.

These are called **solid objects.**





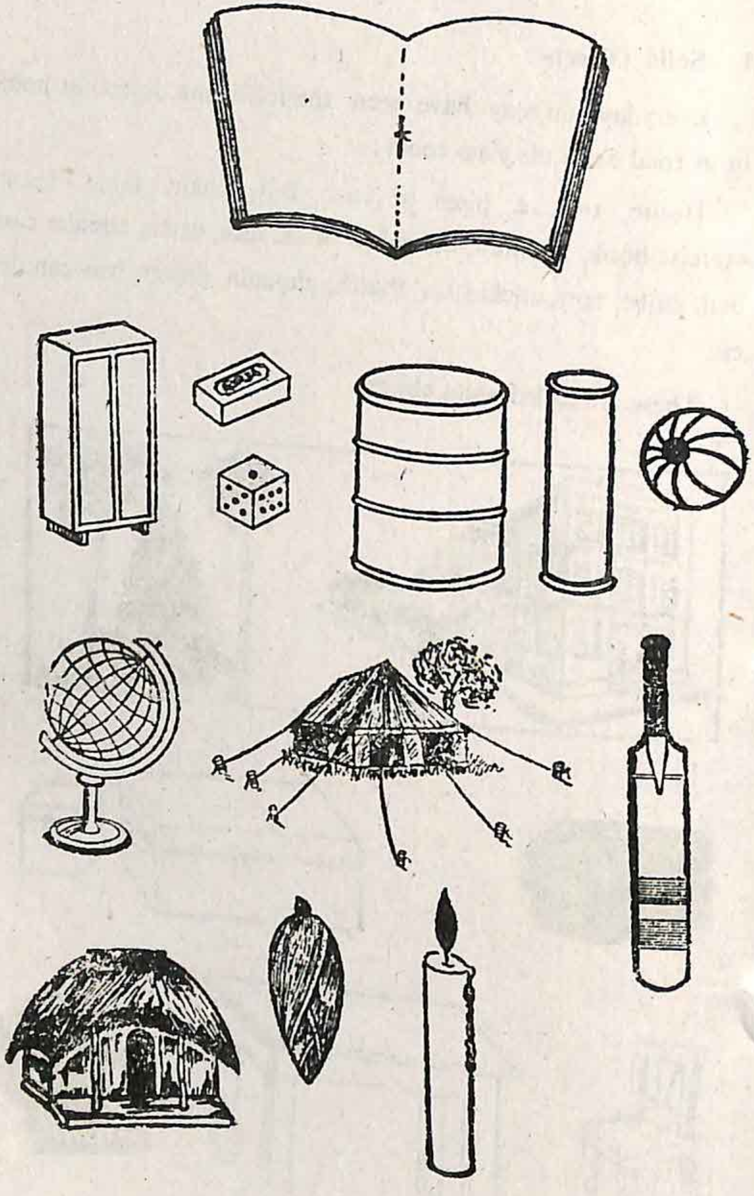
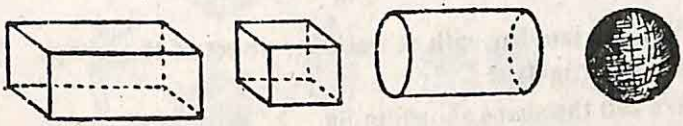


Fig-1

You have seen pictures of many solid objects in figure-1. These solid objects are of different shapes and sizes. We shall discuss here about few special type of solid objects. Before doing this, make a list of solid objects which you have seen in a particular day.

Can you find solid objects of the following shapes (fig-2) in your list ?



(i) (ii) (iii) (iv)

Fig-2

Objects whose shapes are like fig. 2 (i) are called **Rectangular parallelopiped.**

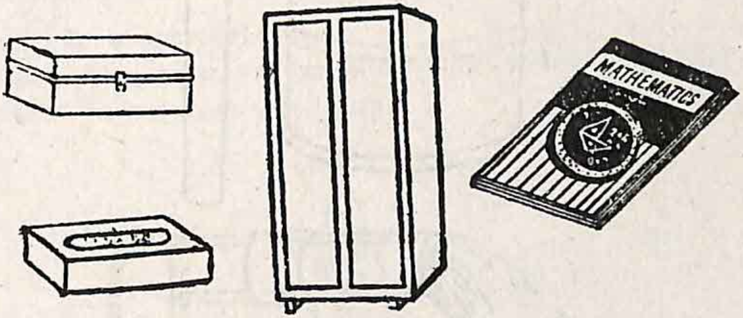


Fig-3

You have seen box, Almilah, book, unbroken brick etc. (fig-3). Shapes of these objects have similarity with that of the rectangular parallelopiped.

Write down the names of some more solid objects which have a resemblance with that of rectangular parallelopiped.

The shape of object shown in fig. 2 (ii) is called cube.

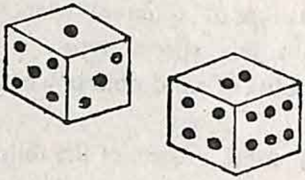


Fig-4

You are familiar with at least one object of this shape. This is ludo-dice (fig-4).

We call the shape shown in fig. 2 (iii) as Cylinder.

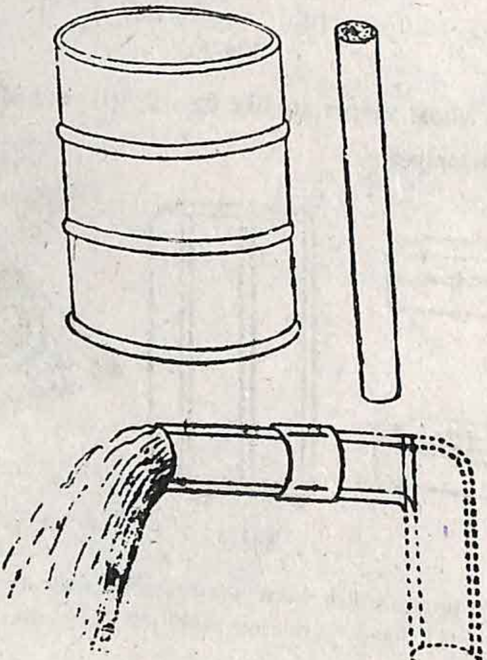


Fig-5

Many of you have seen bamboo pole, trunk of a palm tree or cocoanut tree.. You have also seen **drum of water,** pencil, water pipe (fig-5) etc. All of them are of cylindrical shape.

Now you are in a position to name few more solid objects of such type. We shall call them **Cylinder.**

The object shown in figure 2 (iv) is called **sphere.**

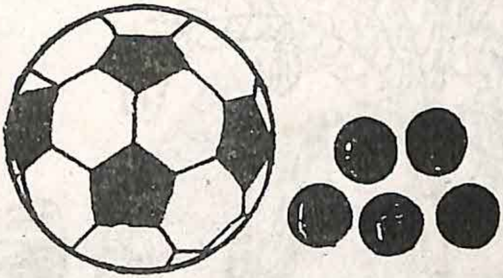


Fig-6

Some objects of this shape are definitely familiar to all of you. Is not it ? Are not these figures reminding you foot ball or marble (fig-6) ?

They are of spherical shape.

Apart from these you are also familiar with different solid objects which you may not see often.

Fig-7

Fig-7

What shape does a cylindrical pencil assume when it is cut by a sharpner (fig-7) ? Have you seen some more objects of this type ?

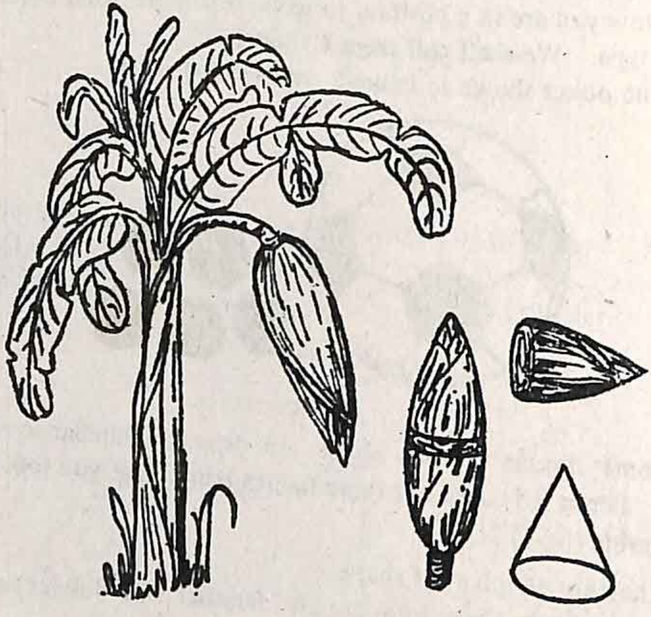


Fig-8

(i)

If you cut out a plantain flower, does not the upper part take this shape (fig-8). The solid objects of these types are called Cone [fig-8 (i)].

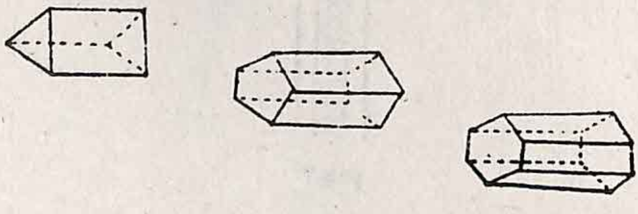


Fig-9

(i)

Many of you have seen pencil with six edges. There are also toys made of glass or plastics which have also more edges. These types of objects are called **Prism** (fig-9).

Remember that a rectangular parallelopiped or a cube is a special kind of prism.

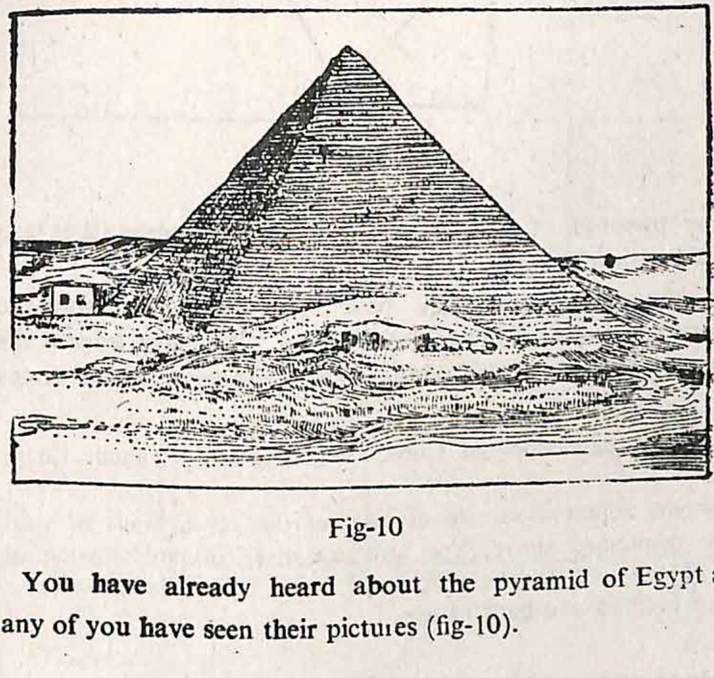


Fig-10

You have already heard about the pyramid of Egypt and many of you have seen their pictures (fig-10).

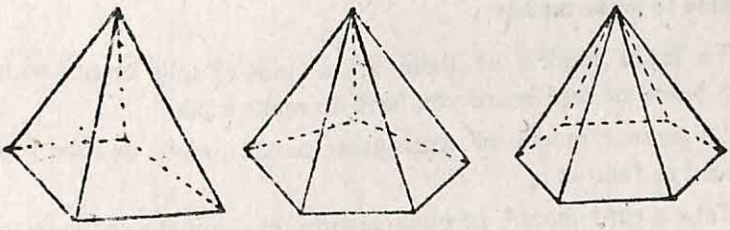


Fig-11

The objects of these shapes are called **Pyramid.**

Few diagrams of a pyramid are shown in fig. 11.

In fig. 12, you can see a special kind of pyramid.

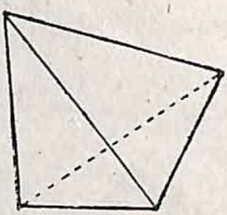


Fig-12

The pyramid of this shape is called **tetrahedron** (that is, a pyramid with four faces).

In above discussion, you have come to know about the different shapes of solid objects. You also have learned the names of few particular kind of solid objects. These names are :

Rectangular parallelopiped, Cube, Prism, Pyramid, Cylinder, Cone, Sphere.

Now you collect a sample of each of the seven kinds of solid objects mentioned above. You also can make or build models of these objects with the help of wood, earth or by Plaster of Paris or even by thick and hard papers.

A guide to make models

To make models of these seven kinds of solid objects with pitch board or card board you have to make a plan.

To make a model of rectangular parallelopiped or cube you proceed as follows :

Take a card board or pitch board. Cut out the shape from a pitch board as shown in fig. 13.

Glue the open ends of this cut-out shape and then fold

Rectangular Parallelopiped

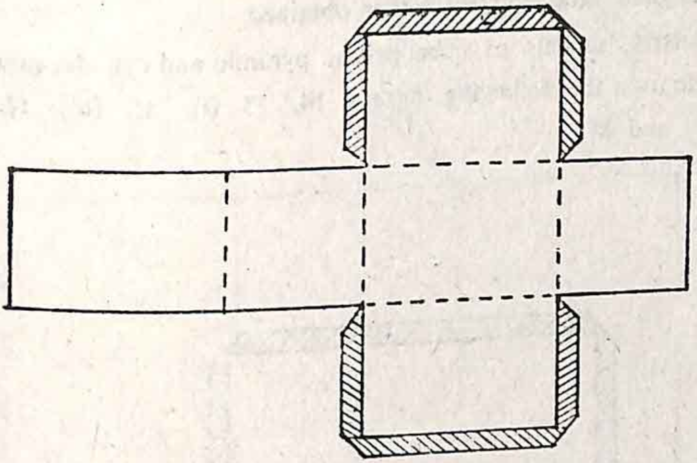
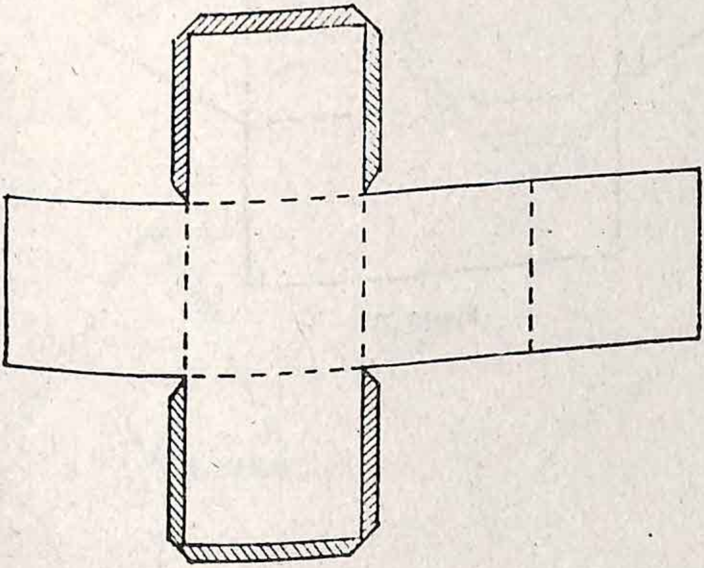


Fig-13



Cube

Fig-14

it along the dotted lines. Finally close the open ends. A model of rectangular parallelopiped is then obtained.

Similarly, models of cube, prism, pyramid and cylinder can be made from the following figures 14, 15 (i), 15 (ii), 16 (i)−(iv) and 17.

Prism

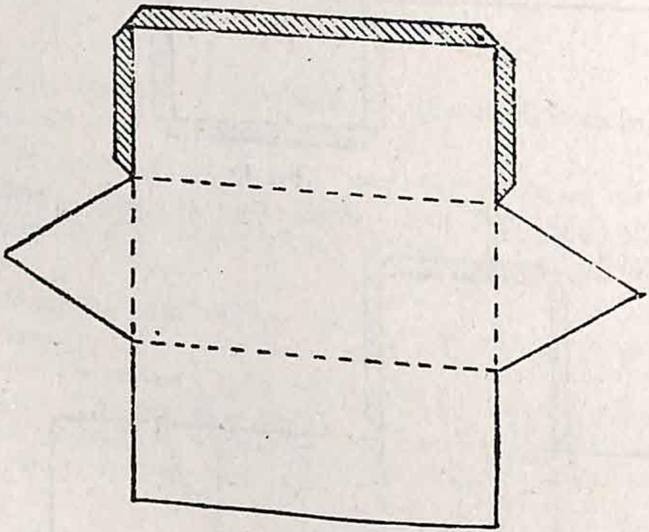


Fig-15 (i)

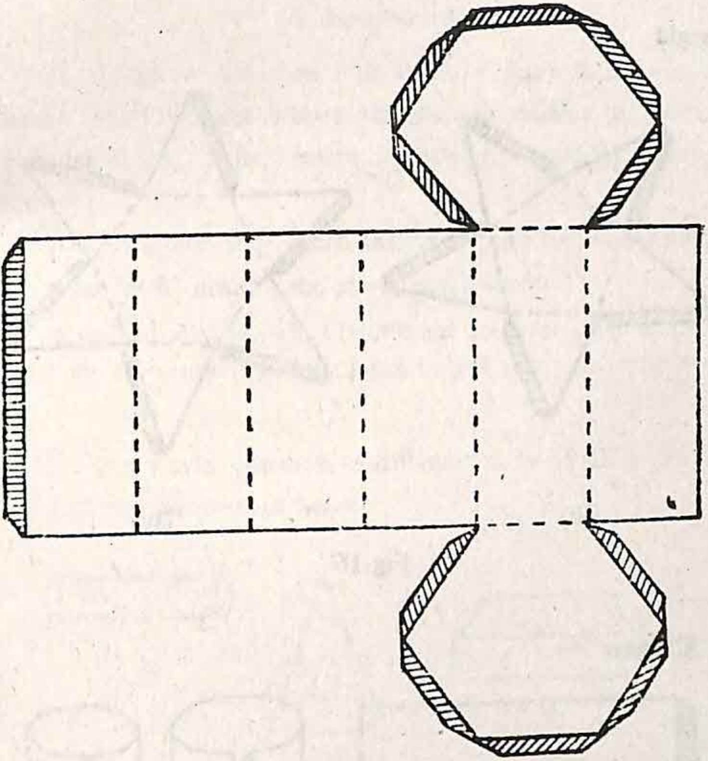
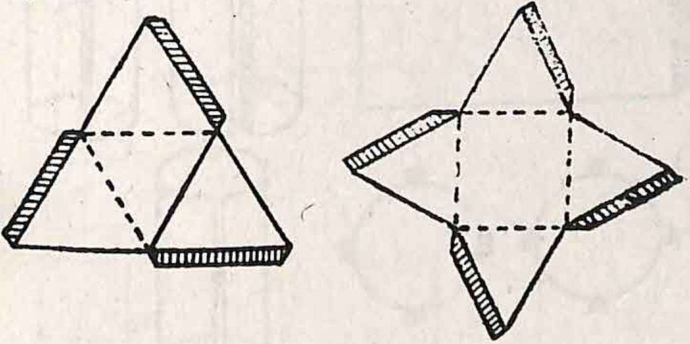


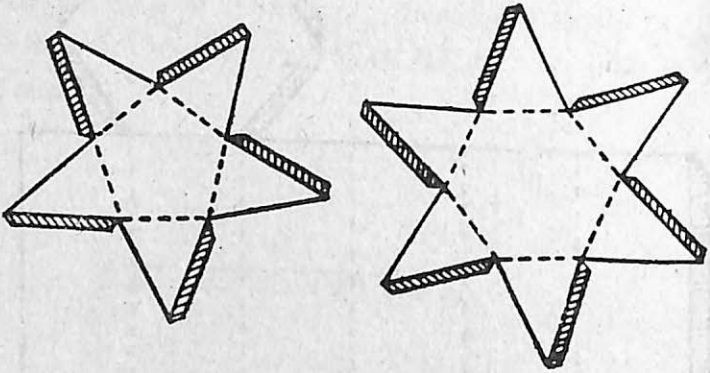
Fig-15 (ii)

Pyramid



(i) Fig-16 (ii)

Pyramid



(iii) (iv)

Fig-16

Cylinder

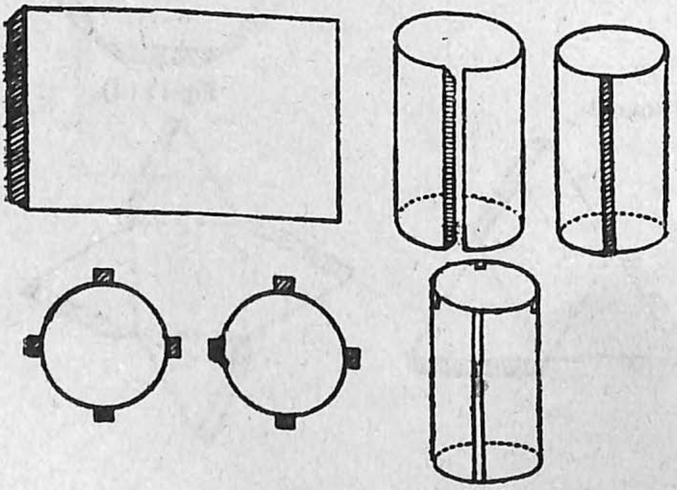


Fig-17

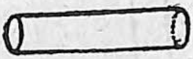
Exercise—1

1. Make a list that will contain one specimen of each of seven solid objects whose shapes are similar to rectangular parallelopiped, cube, prism, pyramid, cylinder, cone and sphere.

Also include five solid objects in your list whose shapes are different from those of the above seven types.

2. Make two clay models of different sizes of each of the seven solid objects mentioned earlier (seek help of your teacher, if required).

3. Draw two pictures of different sizes of the seven kind of solid objects mentioned before.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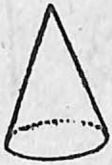
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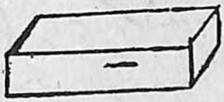
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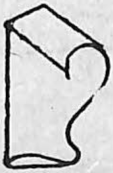
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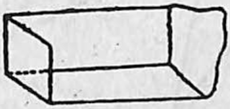
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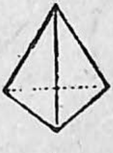
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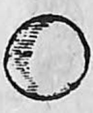
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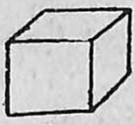
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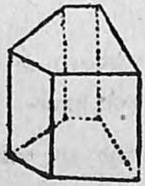
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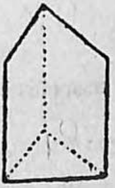
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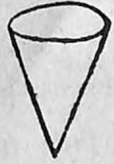
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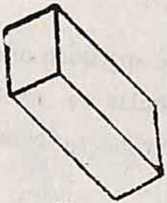
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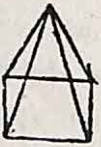
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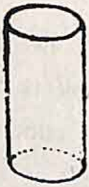
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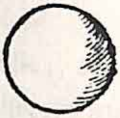
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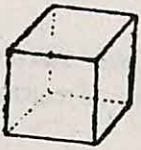
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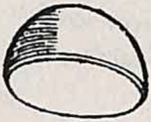
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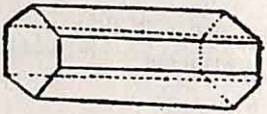
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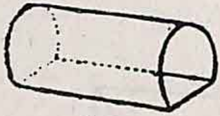
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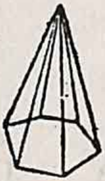
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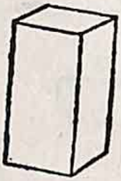
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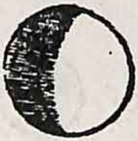
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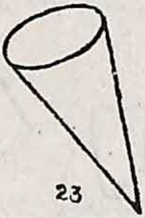
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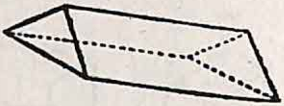

24

4. A collection of figures are shown in the above diagram. A number has been associated to each figure.

(i) From above diagram write the names of the solids in your exercise book at the side of each number in the figure.

(ii) From the above diagram fill up following table :

Shape of the objects	Number associated to the object
Rectangular parallelopiped	
Cube	
Prism	
Cylinder	
Cone	
Pyramid	
Sphere	

2. Plane Figure

Already you have seen solid objects of different shapes and prepared their models also. The surfaces of these objects are of different kinds. If an odject is placed on a paper lying on a table, it is seen that surfaces of some objects will exactly coincide with that of the paper; for example, any face of a rectangular paralleloped. any face of a prism or a pyramid. But curved surface of a cylinder, surface of a sphere will not coincide with the surface of the paper placed on a smooth table. Cosider surfaces of objects which exactly coincide with that of the paper and draw the boundary of these by pencil. One gets then different kinds of figures in different situations. These figures are called **plane figures**.

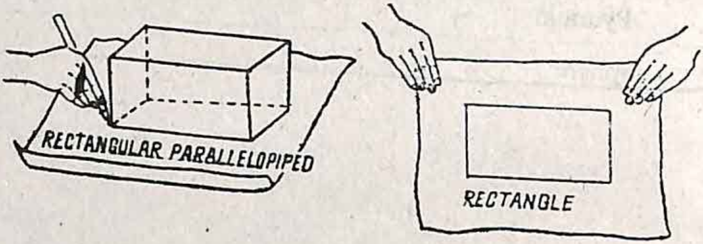


Fig-18

The plane figure obtained by placing a rectangular parallelopiped on a piece of paper on a plane is known as **rectangle** (fig-18).

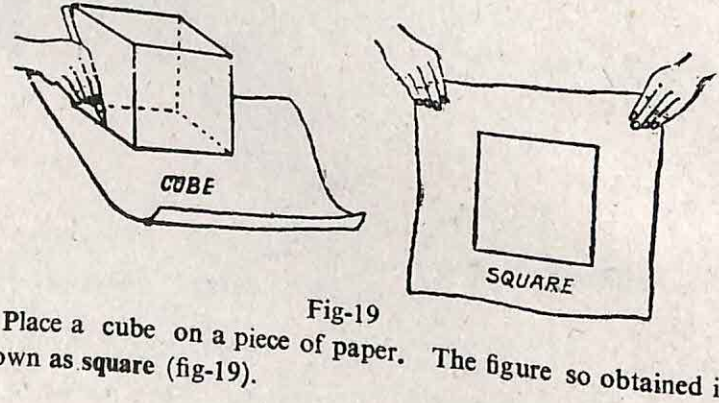


Fig-19

Place a cube on a piece of paper. The figure so obtained is known as **square** (fig-19).

Put the lower face of a pyramid or lower base of a prism on a piece of paper. One thus gets closed figurers known as **polygon.** These figures may be a **triangle** or a **quadrilateral** or a **pentagon** etc.

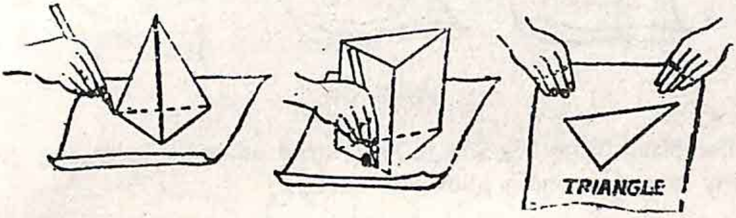


Fig-20

The solid objects in fig. 20 if placed on a plane piece of paper and maked along its boundary, one gets a plane figure, which is a **triangle**.

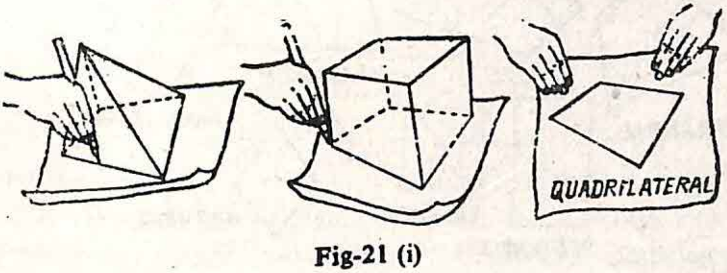


Fig-21 (i)

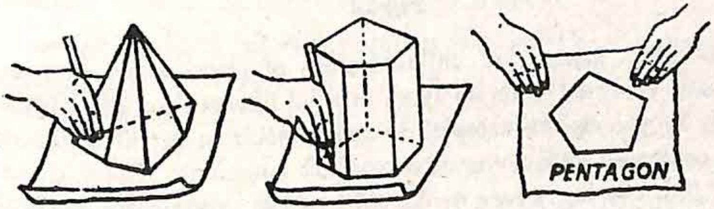


Fig-21 (ii)

Similarly, the figure obtained in fig. 21 (i) is known as a **quadrilateral** and in fig. 21 (ii) as **pentagon.**

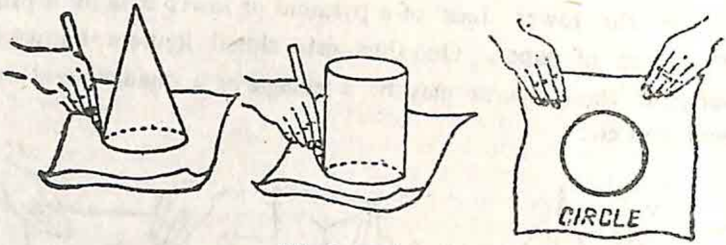


Fig-21 (iii)

The plane figure [fig-21] (iii) objtained on paper by placing the lower face of a cone is known as **circle**.

Apart from these plane figures, you should be familiar with some special types of quadrilateral. These are shown is the fig. 22.

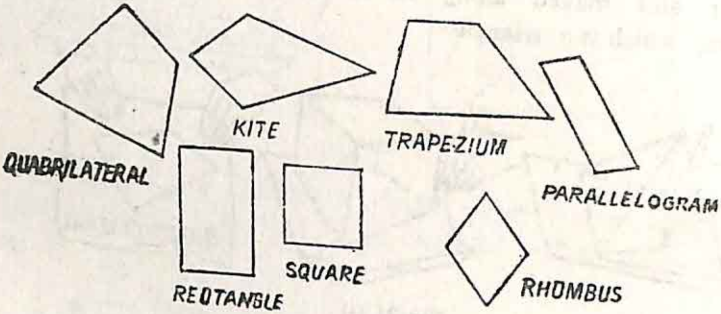
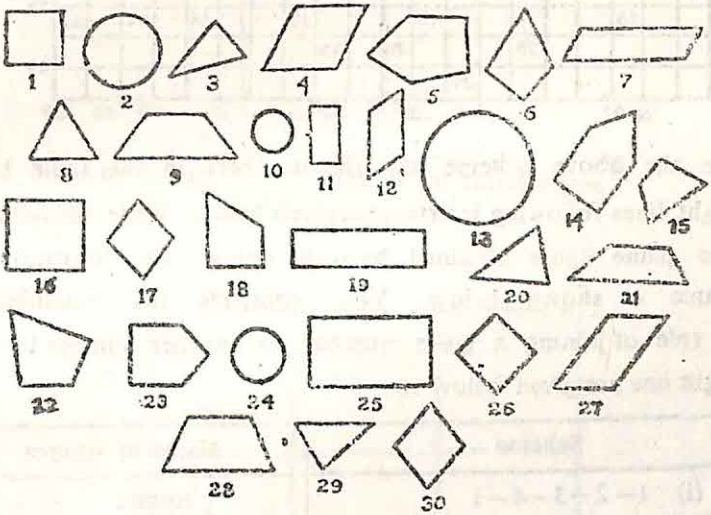


Fig-22

You collect samples of different kinds of plane figures also, as you have collected different types of solid figures. You draw plane figures by placing the samples of solid objects or get these drawn on a cardboard and cutting it along their boundaries. Observe that upper face and lower face of the solid cut out position are identical to the shape of that plane figure.

Exercise—2

1. Pictures of some plane figures are given below. Collect those numbers which are associated to figures of the same shape and write them. Write also the name of the plane figures against such numbers.

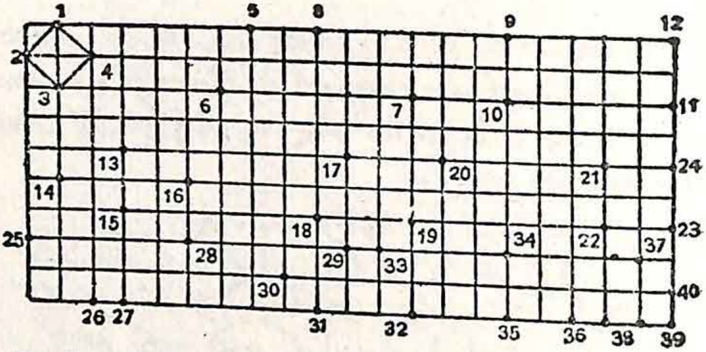


2. From the above figure write out the numbers corrersponding to polygon, quadrilateral, trapezium, parallelogram, rectangle, square, rhombus, triangle and circle.

As for example to the numbers associated to the shape of parallelogram are 1, 4, 7, 11, 12, 16, 17, 19, 25, 26, 27, 30.

3. You make some models of plane figures. Mix these with the models of your friends whe are sitting on the same bench in your class. Before mixing them write your names in the models so that they could be separated easily, Now divide these models among yourself and then identify models of the identical shape and separate them. Test, who can do this more quickly.

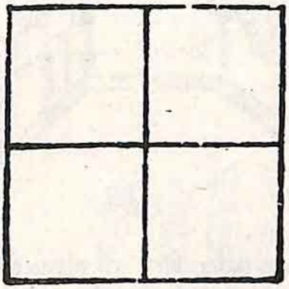
4.



In the above scheme join the numbers in the table by straight lines following instructions given below. Write the name of the plane figure obtained by such joining. An illustrative example is shown below. You complete the remaining. The rule of joining a given number to another number by a straight line are given below :

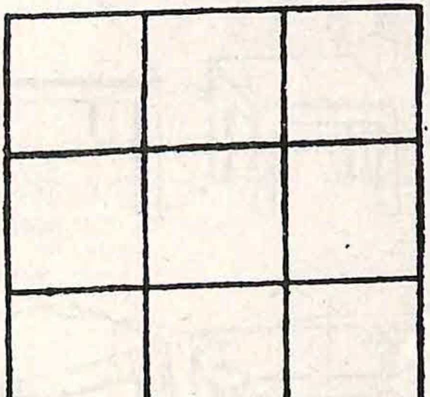
Scheme	Name of figures
(i) 1—2—3—4—1	square
(ii) 5—6—7—8—5	
(iii) 9—10—11—12—9	
(iv) 13—14—15—16—13	
(v) 17—18—19—20—17	
(vi) 21—22—23—24—21	
(vii) 25—26—27—28—25	
(viii) 29—30—31—32—33—29	
(ix) 34—35—36—34	
(x) 37—38—39—40—37	

5(i)



How many squares are there in the above figure ?

5(ii)



How many squares are there in the above figures ?

3. plane

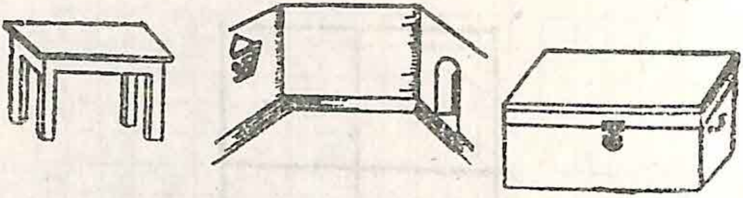


Fig-23

You already have some idea of plane figures, upper portion of a table, the floor of a room, walls of a room or outside or inside faces of a box are plane surfaces and represent some plane figures (ftg-23).

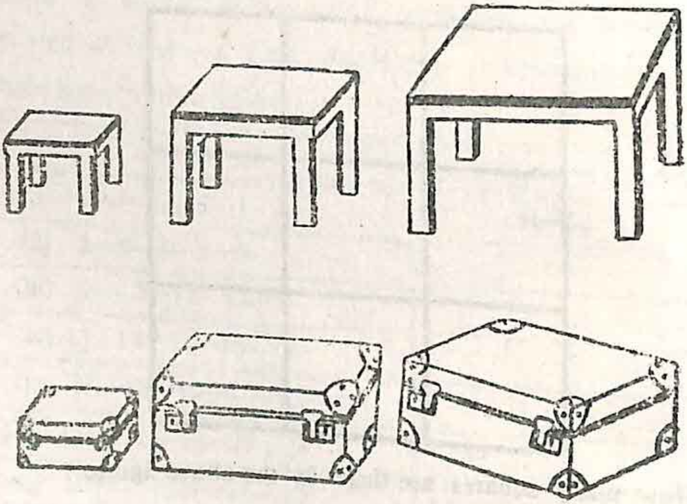


Fig-24

One can extend the length and breadth of a table more and more (fig-24). Each page of your exercise book can also be of bigger size both in length & breadth. There is no end to this proess of extending length and breadth. One can extend it at one's will. Remember that the figuer obtained by extending it in above manner will be called **plane**. The plane figures obtained earlier or the upper face of a table are portion of one or other **plane**.

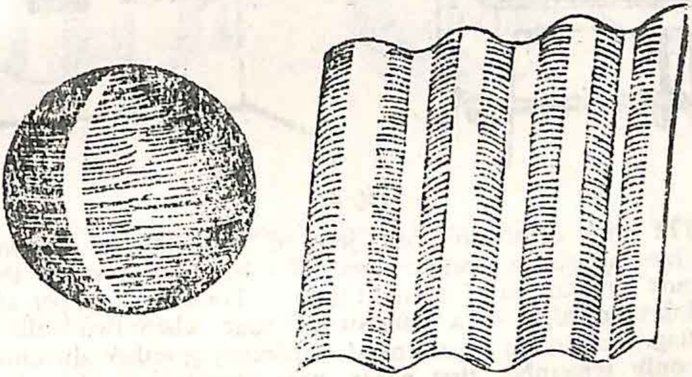


Fig-25

The surface of a football, the upper surface of a tin are not planes (fig-25). These are **curved** surfaces or a portion of a curved surface.

4. Straight lines

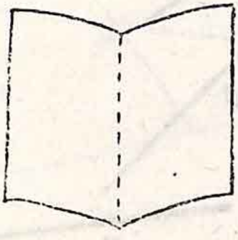


Fig-26

We can imagine any surface of a paper as a plane or portion of a plane. Fold one such paper carefully. The crease, made by such folding may be taken as a straight line (fig-26).

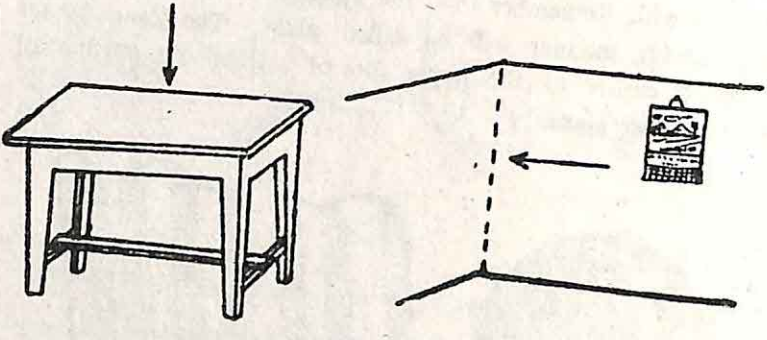


Fig-27

The edge of a table, the join of two walls in a room— all. these illustrate straight lines. To be more precise, these are part or portion of straight lines. Trace on a paper after folding, ,the edges of a table or the edge where two walls are meeting. There could be extended indefinitely in either directions. You only remember that edges here represent straight lines. Straight lines are extended indefinitely in both ways. Place a ruler on a paper and mark it along its edges by a pencil, then a portion of a straight line is drawn. You also can draw a straight line in a better way by a very pointed pencil.

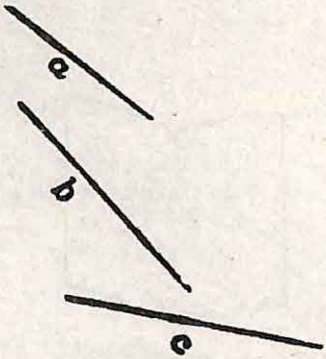


Fig-28

A straight line is represented by small leters a, b, c, .. etc. In fig. 28, three straight lines a, b, c are shown.

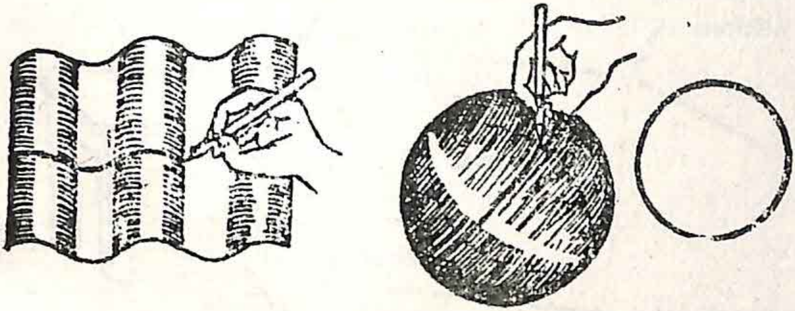


Fig-29

The wavy edge of a corrugated sheet, a trace on a ball are not portion of a straight line, these are curved lines (fig-29). You have seen the figure of a circle. To trace a circle one uses curved line (fig-29).

5. points

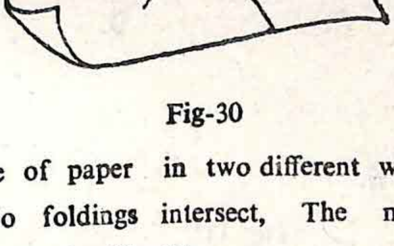


Fig-30

Fold a piece of paper in two different ways so that the creases of two foldings intersect, The meet of these two creases make a point (fig-30).

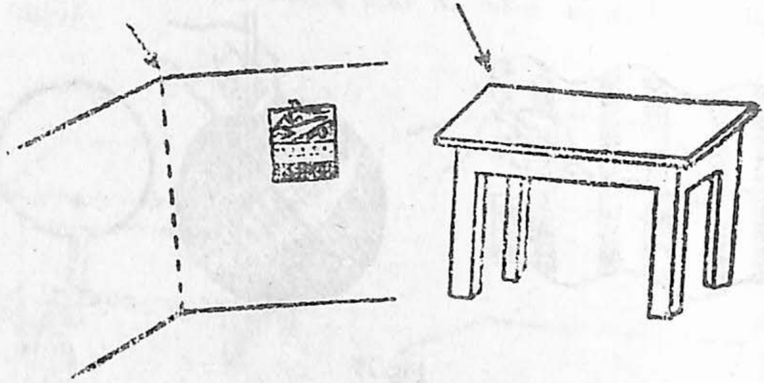


Fig-31

The meet of two adjacent edges of a table, the meet of three walls of a room are examples of points (fig-31).

Fig-32

Make a fine dot on a piece of a paper by a pointed pencil, the dots so obtained will be assumed as points. The smaller a dot will be, the better will be representation of a point (fig-32),

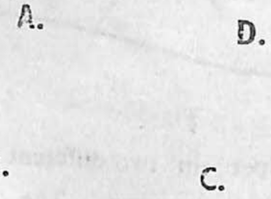


Fig. 33

To distinguish a point from a collection of points the naming of these points are essential. Points are represented by capital letters A, B, C, etc. (fig-33),

Exercise—3

1. From class room situation give some examples to have the idea of point, straight line and plane.

2. Give four eaample of curved line and curved surface.

3. From your collecsion of models of solid objects and plane tigures, exehibiit some samples of plane surface and curved surface.

4. In your exercise book draw three straight lines and three points. Name these lines and points.

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6. Relation between points and straight lines

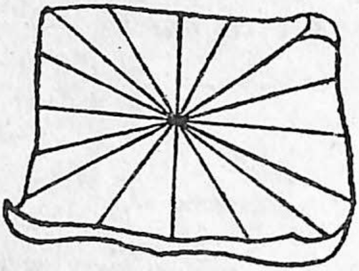


Fig-34

Draw a point on a piece of paper. Fold the paper in such a way that the crease passes through the point. Is not the trace of the folding a straight line ? You make such different foldings repeatedly and will get different straight lines. How many times can you fold the paper ? As many times as you like. is not it ? What do you understand by this ?

Through a point one can draw as many lines as one wishes (fig-34).

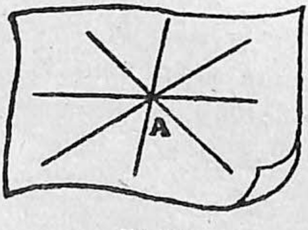


Fig-35

Take a point A on a page of your exercise book. Place a ruler which is very close to the point A. Then you make a trace by a pencil so that the trace line so obtained passes through the given point. Placing the ruler at different position draw different such lines all passing through the given point.

Then you will see that through a point you can draw as many lines as you wish (fig-35).

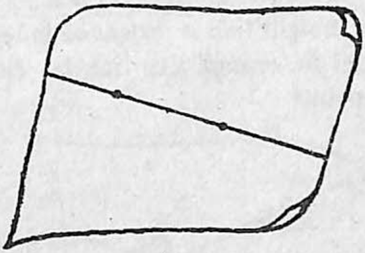


Fig-36

Now take two differnt points on a piece of paper. Fold it in such a manner that the crease passes through two given points (figs36).

Try to make another such folding so that this crease also passes through those to given points. Then you will see that these creases are identical.

Take two points on a page of your exercise book, draw a line by a ruler such that it passes through the given points. If you try to draw another such line, you will see that these two lines exactly coincide. So through two give points one cannot draw more than one straight line. This means :

Through two points one and only one line can be drawn.

So we can now name the straight line by the names of these

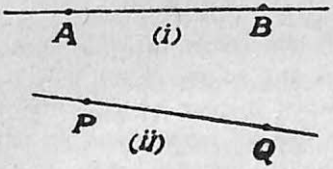


Fig-37

two points. As for example, the line in the figure 37 (i) passes through the points A and B. This straight line is denoted by A B. In figure 37 (ii), P Q is a straight line. To understand that a straight line is extended indefinitely in both directons, it is better to extend the line in either directions beyond those two points.

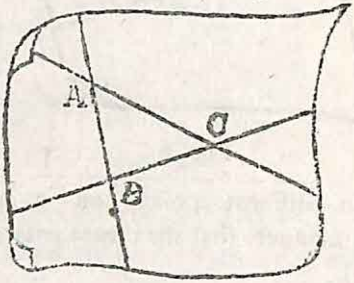


Fig-38

Take three points A, B, C on a paper as shown in fig. 38. Now try to fold it in such a way that these thre points line on a single fold. You will fail to achieve it. The crease can pass through points A and B but not through

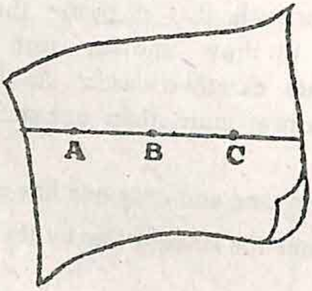


Fig-39

the point C. Similarly the crease can pass through points B and C but not through A and so on. If the position of three points are taken in a particular manner as shown in fig. 39, then one can get a folding so that all three points lie on same crease. So three points on a plane, may or may not lie on a straight line.

You can also verify that one can not always draw straight line through more than three points. So we come to the conclusion:

Three or more than three points may or may not lie on a straight line.

When three or more than three points lie on a straight line, we call them collinear points.

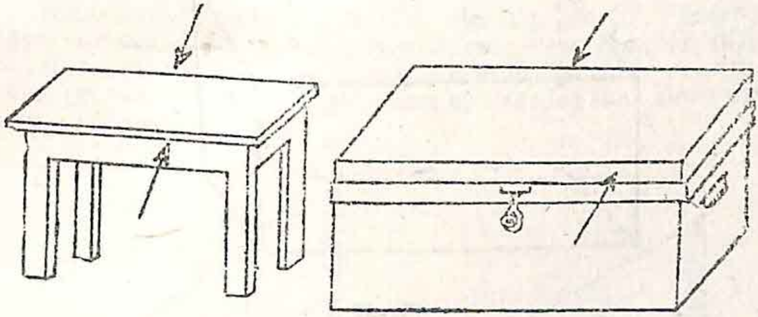
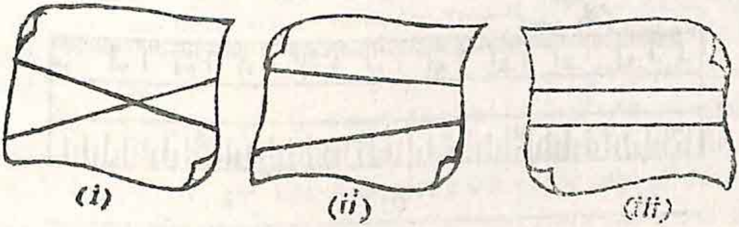


Fig-40

Observe that two adjacent edges of a table or two adjacent edges of the upper face of a rectangular box meet at a point. Do the opposite edges meet? The obvious answer is no. Will they meet if opposite edges are extended indefinitely in either direction? No, they will not meet (fig. 40). You can verfiy this fact by paper folding or by drawing straight lines on the page of your exercise book.



(i) (ii) (iii)

Eig-41

Remember that the straight lines that you have drawn are in fact, parts of straight lines. In figure 41, observe that in the first diagram two straight lines are drawn which interest at a point. In the second diagram, two straight lines do not intersect, but they will intersect at a point if extended. In third diagram, two straight lines will never intersect at a point even if they are produced in either direction. So we observe:

Two straight lines lying on a plane will not interesect always.

If the two lines intersect they have only one common point and the point is called **point of intersection.**

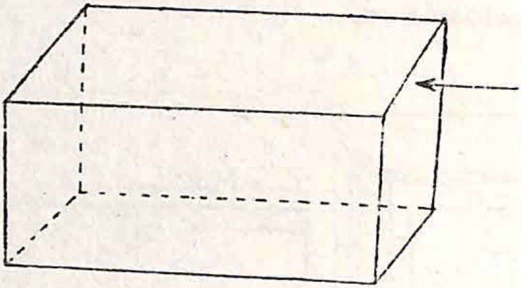
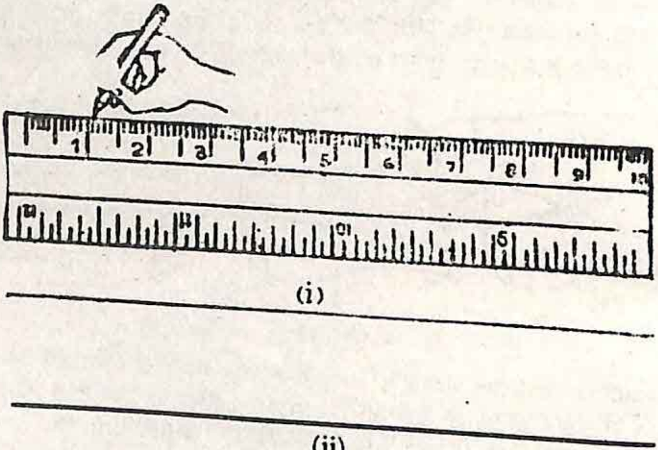


Fig-42

If the two lines do not intersect even if they are extended we call these two lines as **parallel** to each other.

Two opposite edges of the floor of a room, two opposite edges of the some face of a wall are examples of parallel straigt lines.



(i)

(ii)

Fig-43

In figure 42, one can see that one edge along the breadth of the upper face of the box and other edge along the length of the lower face do not meet. But these two edges are not parallel as these two edges are not lying on the same plane.

By parallel straight lines we shall mean those lines which lie on one plane and which do not intersect one another.

Draw lines along two edges of a ruler [fig. 43 (i)]. Extend these two straight lines, but they will never meet [fig. 43 (ii)]. So, it is seen that these two straight lines are parallel. we can, thus, get two parallel straight lines by drawing lines along the opposite edges of a ruler.

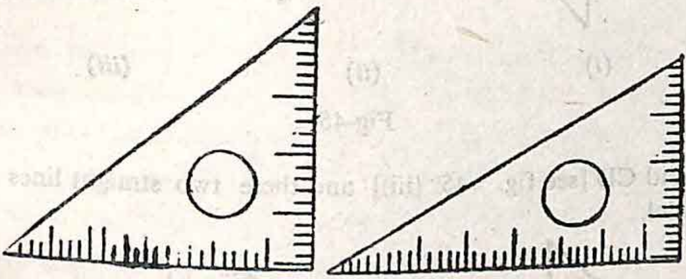
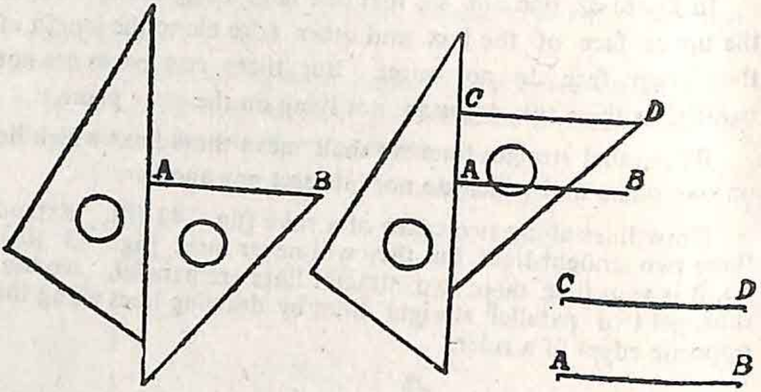


Fig-44

To draw parallel straight lines we use instruments. The name of one such instrument is set-square. In figure 44, you are seeing two set-squares. In your geometry box, there are two such set-squares. You take out these and observe the two adjacent edges of these instruments. These two edges are so located as if one stands erect over the other. We shall call any one of them as **erect edge.**

As in figure 45 (i) place an erect edge of one set-square pressing close to one edge of the other. Fix these by one hand and draw a straight line A B along the upper edge of the set-square on the right. Keep the left one fixed and move the right one either in uqward or in downward direction.

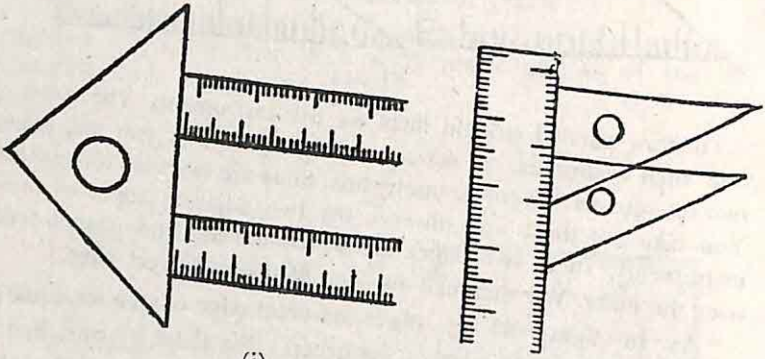
The fig. 45 (ii) exhibits the position when the right one is moved in the upward direction. In this position draw the straight line C D along the upper edge of the set-square on the right. Remove these set-squares. You get two straight lines



(i) (ii) (iii)

Fig-45

AB and CD [see fig. 45 (iii)] and these two straight lines are parallel.



(i) (ii)

Fig-46

Using a set:square and a ruler you can also draw two parallel straight lines as drawn in figures 46 (i) and 46 (ii).

You already know that through a point one can drew many straight lines (fig. 47).

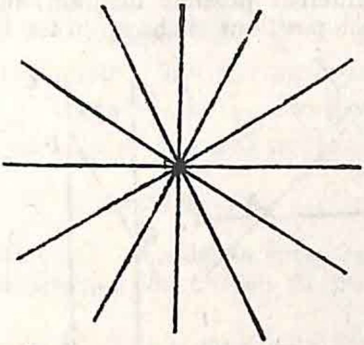
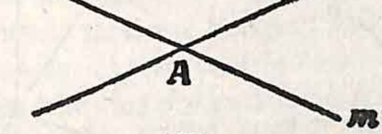


Fig-47

Let us draw a number of straight lines on a plane. Will all these straight lines meet at a single point ? You already know that two straight lines on a plane are eithor parallel [fig. 48 (i)] or meet at a point [fig. 48 (ii)]. In fig. 48 (i) to lines a and b are shown to be parallel and in fig. 48 (ii) two lines l and mare shown to be intersecting at a point A.



(ii)

Fig-48

We now take three straight lines on a plane. Let us try to investigate the different positions in which the lines can lie on the plane. All possible positions are shown in figure 49.

Try to draw different possible diagrams and verify that these are only four such positions as shown in the figure.

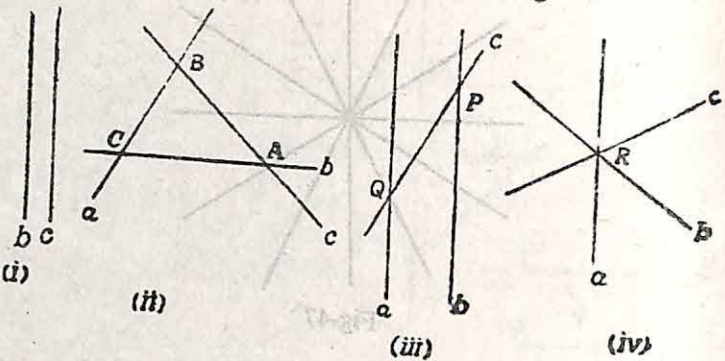


Fig-49

In fig. 49 (i), three lines a, b, c are drawn and they are not intersecting one another.

In fig. 49 (ii), one line intersects the other two lines. Also every pair of lines intersect at distinct points.

In fig. 49 (iii), the line c intersects the two lines a and b at the points P and Q respectively.

All the three lines a, b, c meet at a single point R as shown in fig. 49 (iv). Here we say that the three lines are **Concurrent.**

From above figures it is clear that any three lines lying on a plane may or may not be **Concurrent.**

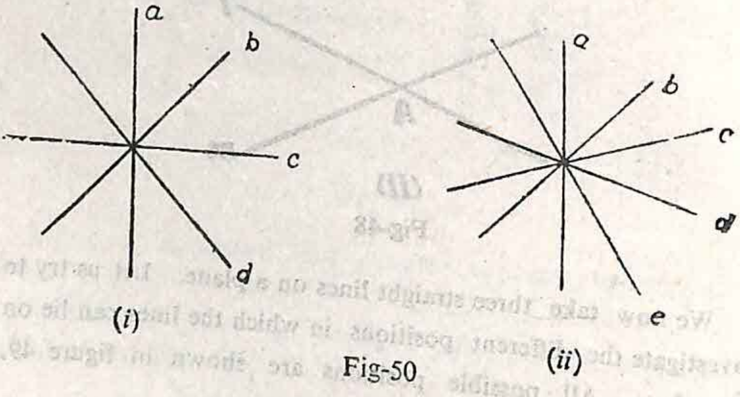


Fig-50

In fig. 50 (i), four straight lines a, b, c, d are concurrent while in fig. 50 (ii) five lines a, b, c, d, e are concurrent.

From above two figures we can come to the conclusion that three or more straight lines may or may not be concurrent.

Exercise—4

1. Take a point A. Through A draw four straight lines. Name these straight lines. How are these straight lines situated ?
2. Set two points A, B on a plane. Join A and B by straight line. What is the name of this straight line ? Take another pint C on this straight line. What is the name of the points points A, B, C ?

   Take a point D on the plane not lying on the above straight line. Draw the straight lines AD, BD, CD.
3. Draw the straight line AB. Take three points C, D, E on the straight line. What is the name of this collection of five points ?
4. (i) Draw two parallel straight lines with the help of set-squares.

   (ii) Draw two parallel straight lines with the help of a set-square and a ruler,

   Draw a straight line XY. Take a point P lying outside the straight line. Through P draw two such straight lines which intersect the given line XY.

   Give names to these two points of intersection. Write down the name of these straight lines.
6. Fill up the following blanks choosing appropriate word or phrase given at the end of each incomplete statements.

   (i) Through a given point——–straight line can be drawn.

   (one, two, many)

   (ii) At least——–are essential to draw a definite straight line.

   (one point, two points, three points)

(iii) If a straight line passes through three given points, these points are called———.
(plane, collinear, concurrent)

(iv) If two straight lines, lying on a plane, are not parallel, they intersect at———.
(one point, two points, many points)

(v) If two straight lines lying on a plane do not intersect, they are called———.
(collinear, concurrent, parallel)

(iv) The total number of lines passing throngh all the four points which are not collinear is———.
(zero, one, four)

(vii) The total number of lines passing through four collinear points is———.
(zero, one, four, six)

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7. Line Segment

Place a rectangular parallelopiped on a piece of paper. Draw line along one of its edges from one end to the other fig. 51 (i)].

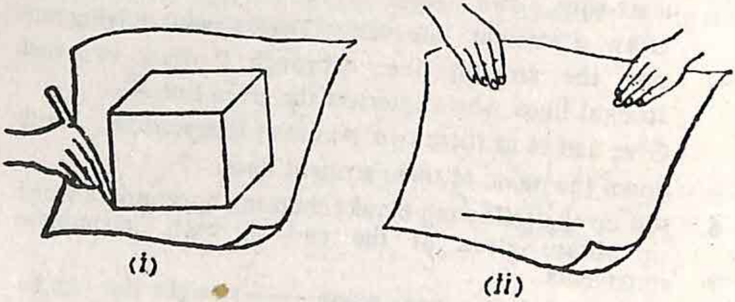

(i) (ii)

Fig-51

You already know that the figure [fig. 51 (ii)] drawn on the paper is a part of a straight line. This figure is known as **line segment.** A line segment has two end points.

Fig-52 (i)

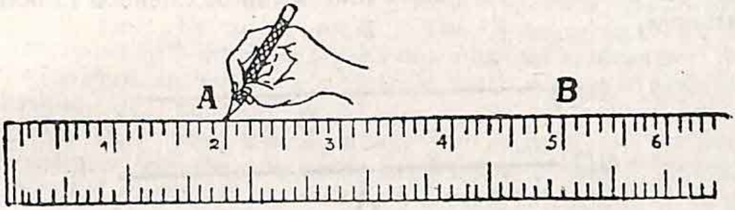


Fig-52 (ii)

Fig-52 (iii)

Take two points A and B [fig. 52 (i)]. Place a ruler in such a way that these two points A and B become very close to one edge of it. A and B are then joined [fig. 52 (ii)]. We thus get the line segment A B with A and B as its end points [fig. 52 (iii)]. When a figure of a straight line is drawn it is usually seen that the figure has two end points but we should realise that the straight line is not terminating at the two end points, it is extended in both the directions. By the line segment AB, we mean that portion of the straight line which is between A and B. When we speak of the straight line A B, it is meant that A and B are not end points and A B is extended in either directions.

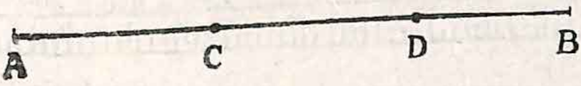


Fig-53

See the above figure (fig. 53). Line segment AB, line segment CD, line segment AD etc. are distinct though they all are portions of one and the same straight line. But straight line AB, straight line CD and staright line AD are not distinct. They all represent one and the same straight line.

You have noted that a straight line has no ends and so there is no question of extending it in any direction ; but a line segment has end points and it can be extended in both directions.

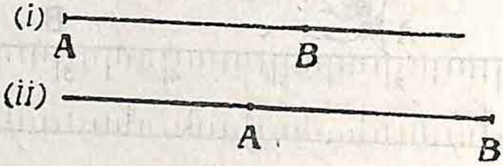


Fig-54

The line segment AB is shown to be extended in the direction from A to B in fig. 54 (i). In fig. 54 (ii) line segment AB is shown to be extended in the direction from B to A. In first figure, it will be said that the line segment AB is extended and in the second figure it will be said that the line segment BA is extended.

Now you have come to know that a line segment has two end points. The distance between these two end points is called the length of the line segment. Naturally, the concept of distance cannot be associated to a straight line as a straight line has no end points.

You all are familiar with the instrument which measures the length of a segment. We call this ruler or scale.

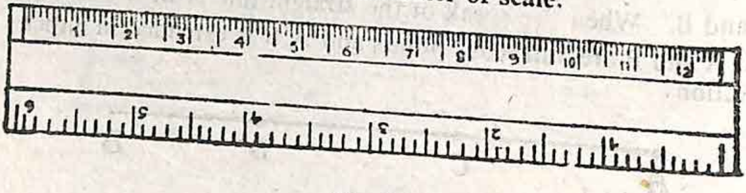


Fig-55

The figure-55 shows a ruler. Both edges of the ruler are marked. Some of these markings are big and some of them are small. The numbers 1, 2, 3, 4, etc. are written at the side of big marks on both sides of the scale. Along one side of the ruler markings are close in comparison to the other. This side indicates centimetre scale and otherside indicates inch scale. The distance between the initial mark and mark 1, or marks 1 and 2, or 2 and 3, or 3 and 4 etc. in the instrument is known as one centimetre. Each centimetre is again divided by 10 parts by small marks. The distance between the consecutive such small marks is known as one millimetre. For convenience of counting a middle mark is given at each fifth division.

We shall now discuss about the measurement of a line segment. Place the ruler along line segment in such a way that the initial mark of it becomes very close to one end point of the line segment. Adjust the ruler in such a way that centimetre side of the scale is very close to the entire line segment. Now observe which mark of the scale coincides with the other end point of the line segment. The distance between these two marks of the scale is the length of the line segment.

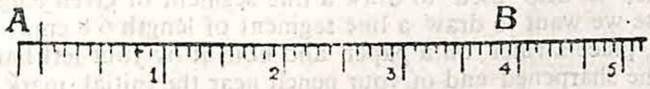


Fig-56

In figure-56, you observe that the end point A coincides with the initial 0 mark of the scale and other end point B coincides with the 5th small mark after the big mark 4. So, the length of the line segment is 4 centimetre 5 millimetre or 4·5 cm.

It may be the case that the second end point does not exactly coincide with any one of the marks but may coincide with a point lying between two marks. In this circumstance, the mark which is close to the second end point is taken and corresponding length is measured.

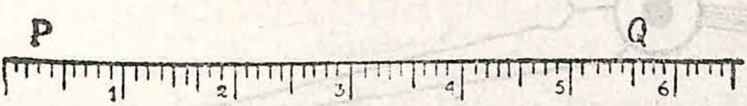


Fig-57

As for example, in figure 57 the point P of the segment

PQ coincides with the initial mark of the centimetre scale and other end point Q coincides with a point lying between mark 5 and mark 6 after the big mark 5. It is seen that the Q is close to mark 6 than the mark 5. Therefore, the length of PQ is assumed to be 5 centimetre 6 millimetre or 5·6 cm.

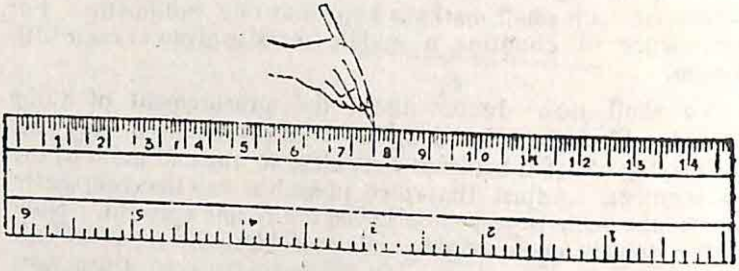


Fig-58

Ruler is also used to draw a line segment of given length. Suppose we want to draw a line segment of length 6·8 cm. To do this, place a ruler on a paper and hold it by your left hand. Place the sharpened end of your pencil near the initial mark of the scale and draw the line up to the point where it coincides with mark of 6 centimetre 8 millimetre. The line segment so obtained has the length 6·8 cm.

To measure length of a line segment or to draw a segment of definite length we take the help of an instrument other than the scale. This is known as **divider**. In has two arms of equal length (fig. 59).

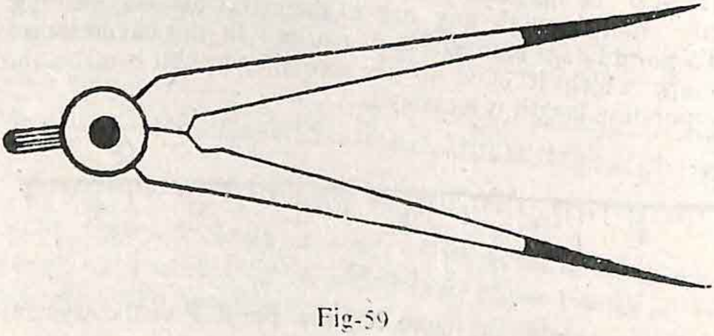


Fig-59

At the end of each arm there is sharp metalic end and other ends of these arms are screwed in such a way that the gap between these two arms can be adjusted.

AB is a line segment. To measure its length sharp end of one arm is placed on A and that of the other arm is placed on B. Remove the divider carefully so that the gap between these two arms is not affected. Place the end point of one arm close to the inital mark of a scale and see the mark which is very close to the end point of other arm. Now you measure the distance between these two marks of the scale. This distance gives the length of the segment. To draw a line segment of length 5.7 cm. by a divider we proceed as follows. Place one end of an arm of the divider with the initial mark of the scale and the other end of the arm with the mark of 5·7 cm. Place this divider on a piece of paper and press it so that the two end points of arms penitrates the paper and make two small holes. Draw a line segment between these two small holes by a ruler. This is the required segment of length 5.7 cm.

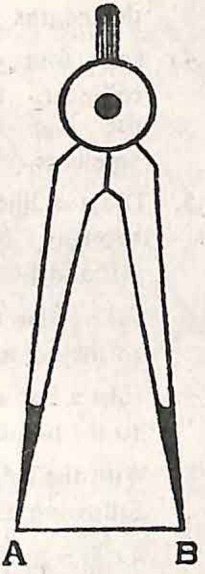


Fig-60

(The more accurate measurement of a segmeut is achieved if it is done by a divider).

Exercise—5

1. Take two points A and B. Draw the line segment AB. Measure its length.
2. Take two points P and Q. Draw the straight line PQ.

3. Take three non-collinear points A, B, C. Draw the line segments AB, BC and CA respectively. Measure the lengths of these line segments.
4. Take four points P, Q, R, S no three of which are collinear. Draw the line segments PQ, QR, RS, SP, PR and QS. Measure the length of these line segments.
5. Draw a line segment AB. Take a point C on this line segment. Measure the length of the line segments AB, AC and BC.
6. Take a line segment AB. Extend line segment AB up to the point D.
7. Take a line segment PQ. Extend line segment QP up to the point S,
8. With the help of a ruler draw line segments of the following lengths :

   4 cm, 6 cm. 9 cm. 2 cm, 4·5 cm, 5·5 cm, 7 cm, 5 mm, 8 cm, 2 mm, 9·1 cm, 2·7 cm, 1·9 cm,. 4 cm, 3 mm, 10·4 cm, 3 cm 8 mm, 6·7 cm, 12 mm, 24 mm.
9. Draw ten line segments of different length and measure them by ruler.
10. Draw the line segments of the following length with the help of a divider :

    9·1 cm, 8 cm. 2·9 cm. 7 cm, 3 cm, 45 mm.
11. Draw eight line segments. Measure these length by a ruler and also by a divider.
12. Draw a line segment of length 12 cm. Divide them into 4 equal parts by a ruler. Verify this division by a divider.

8. Determination of vertices, edges and faces of rectangular parallelopiped, cube and tetrahedron

Already you have made models of rectangular parallelopiped. cube and tetrahedron. Make three models of each kind of above objects of different sizes

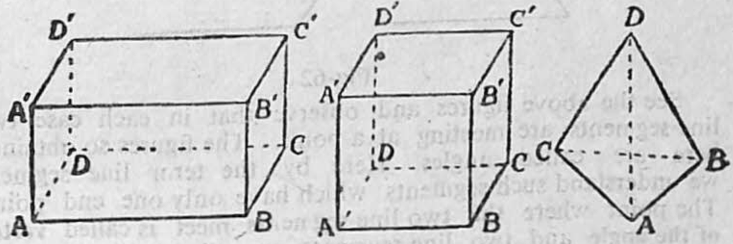


Fig-61

The plane surfaces of those solid objects are called **faces.** If two faces meet along a line segment that line segment is then called **edge.** The point where two or more edges meet is known as **vertex.**

Shape of the solid object	Number of faces	Nnmber of edges	Number of vertices
1. Rectangular parallelopiped			
2. Cube			
3. Pyramid tetrahedron			

Determine the number of faces, edges and vertices of the: above three solid objects and write them in the above table: Now add the number of vertices and the numbe of faces of each of those solids. Then you subtract the number of edges from it. Find what number is obtained in each of these case.

9. Angle

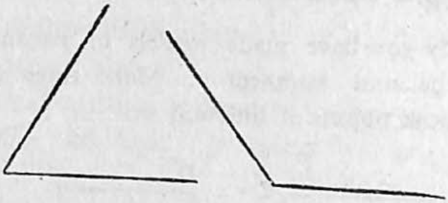


Fig-62

See the above figures and observe that in each case two line segments are meeting at a point. The figures so obtained here are called angles. Here by the term line segment we understand such segments which have only one end point. The point where the two line segments meet is called **vertex of the angle** and two line segments are called **sides (or arms) of the angle.** Take a thread, keep one of its ends fixed and keeping it tightly stretched move the other end, or keep one end of a straight fine wire fixed and move the other end. As a result one gets figures as shown in figure 62 (i).

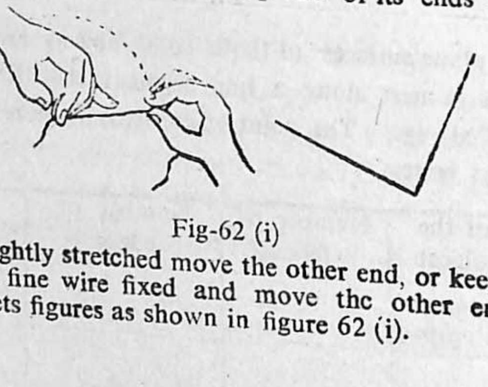


Fig-62 (i)

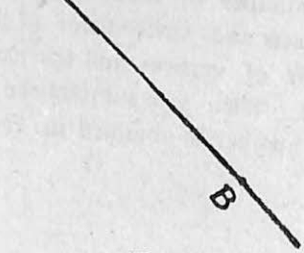


Fig 63

In figure 63, the line segments AB and AC are shown to have met at a point A. Here A is vertex of the angle and AB, AC are its sides. This angle is written as angle BAC or angle CAB. Instead of the word 'angle' the symbol '∠' is also used and one writes ∠BAC or ∠CAB.

By ∠BAC or ∠DAC we mean the same angle as shown in figure 64.

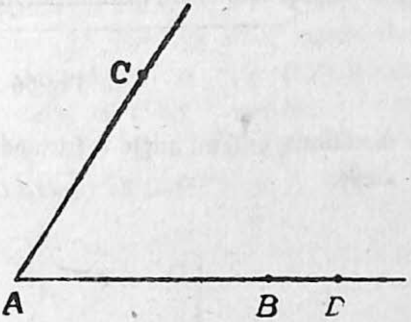


Fig-64

Fix the ends of two sticks or two hard pieces of pitch board by a screw so that these two sticks or the pieces of pitch board can move about the fixed point Keep one of them fixed and rotate the other in anticlockwise direction. You

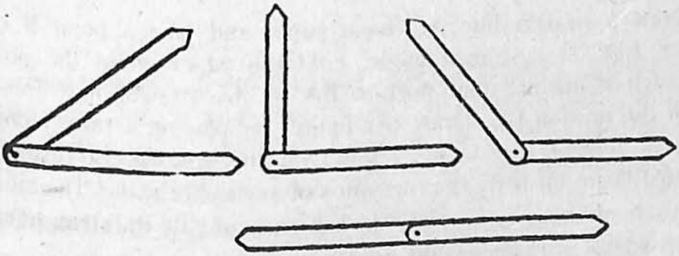


Fig-65

will see that the gap between these two is increasing. This means that the measure of the angle is increasing (fig. 65). Now place the frame made by sticks or pieces of pitch board in such a way that one arm coincides with one of the edges of a scale. Rotate the other arm until it also coincides the edge (see fig. 66). Then both arms of the frame are in oppsite

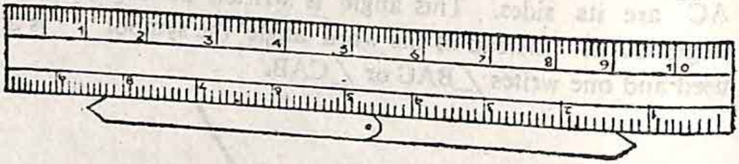


Fig-66

directions, and an angle is formed. This angle is called **straight angle.**

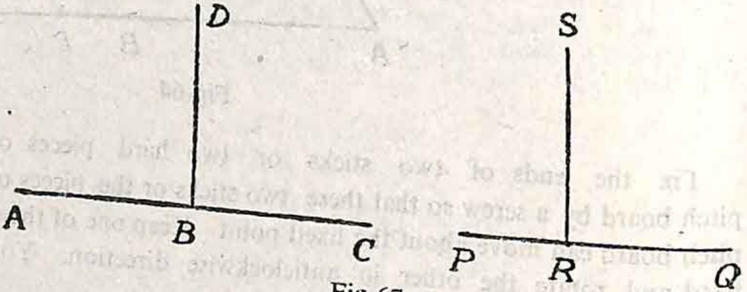


Fig-67

Draw a straight line AC on a paper and take a point B on it. Then ABC is a straight angle. Fold the paper about the point B in such a manner that portion BA of the straight line coincides with the portion BC. Draw the figures by placing a ruler along the line of folding. Let D be a point on this line. As a result ∠CBD and ∠ABD are formed. The measures of angles are same. The measure of each of these two angles is half measure of the **straight angle**. Such angles are called **right angles.**

In whichever way straight angles are drawn, their measures are always same. Each of the right-angle is also of the same measure and is half the measure of the straight angle. Look at the above figure and see that the line segment BD stands erect on the straight line AC and line segment RS also stands erect on the straight line PQ (fig. 67).

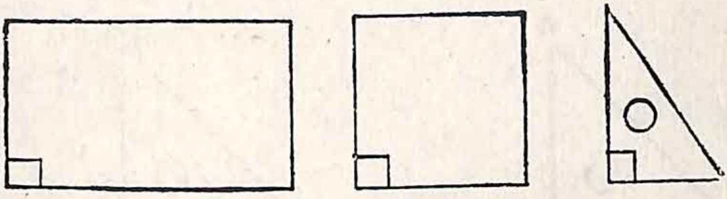


Fig-68

If two adjacent sides of a rectangle are seen, it is found that one stands erect on the other (fig. 68). In other words, two such sides have formed a right-angle. A right-angle is also formed if one consider two adjacent sides of a square or two erect sides of a set-square (fig. 68). In a right-angle one side is standing on the other in perpendicular direction.

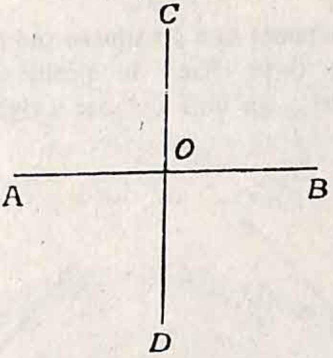


Fig-69

Two straight lines AB and CD intersect each other at the point O in such a way (see fig. 69) that the ∠AOC is a right-angle. We shall say that the line AB is perpendicular to CD or line CD is perpendicular to AB. Note that the angles ∠COB, ∠BOD and ∠DOA are all right-angles.

You also note that ∠AOB is a straight angle. Now one can say that two right-angles ∠AOC ann ∠BOC are foming this straight angle. With the help of two set-squares or by a ruler and a set-square a right-angle can be drawn.

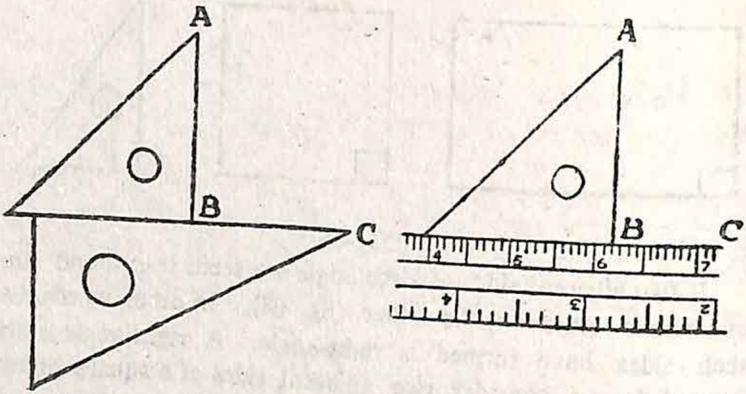


Fig-70

Place two set-squares or a set-square and a ruler as shown in figure 70. Keep them fixed in position and draw lines along AB and BC. In both the case a right-angle ∠ABC is formed.

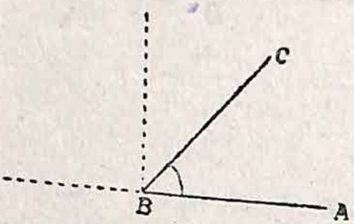


Fig-71 (i)

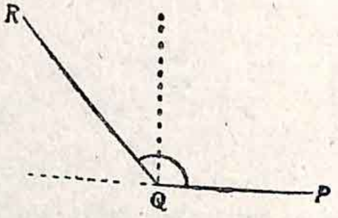


Fig-71 (ii)

In figure 71 (i), it is seen that ∠ABC is smaller than a right-angle but in figure 71 (ii) ∠PQR is bigger than a right-angle but less than a straight angle. An angle which is less than a right-angle is known as **acute angle.** Thus ∠ABC is an acute angle. An angle more than a right-angle but less than a straight angle is called an **obtuse angle.** Thus ∠PQR is an obtuse angle.

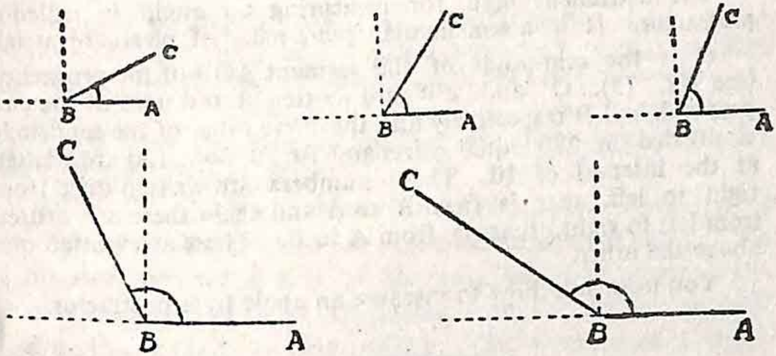


Fig-72

From above figures (fig. 72), it is evident that acute and obtuse angles may be of different measures. How could you ascertain the measure of each such angle ?

We know all right-angles are of the same measure. Naturally the measure of a right-angle can be taken as unit in the measurement of angles. For convenience we divide a right-angle into 90 equal parts. One such division of a right-angle is called one degree. If an angle equals to the sum of ten such divisions, we say that the measure of the angle is ten degree or 10°. Then one right-angle=90° One straight angle=two right-angles=180°.

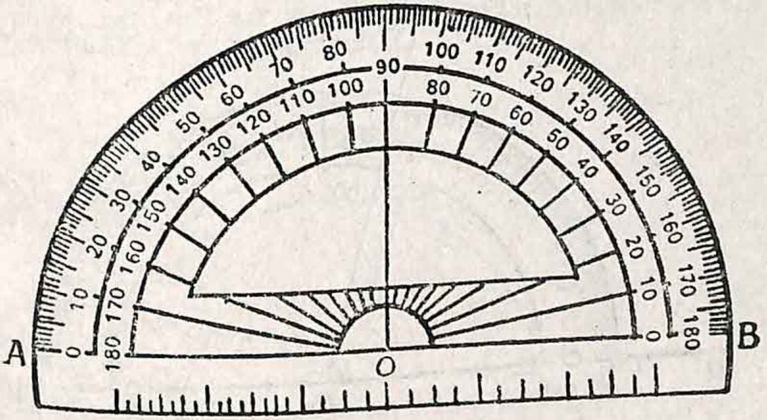


Fig-73

The instrument used for measuring an angle is called a **protractor.** It is a semicircular plate made of plastic or metal.

O is the mid-point of the segment AOB of the protractor (see fig. 73), O° and 180° are written at the sides of the end points A and B respectively and the curved line of the semicircle is divided in 180 equal parts and 10, 20, 30,...180 are written at the interval of 10. These numbers are written once from right to left, that is from B to A and again these are written from left to right, that is from A to B. These are written one above the other.

You now note how to measure an angle by a protractor.

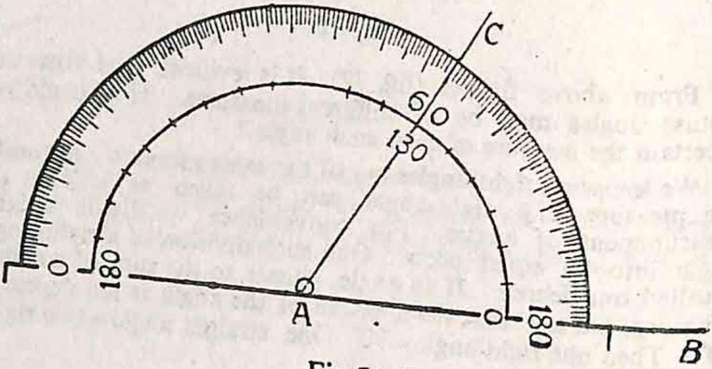


Fig-74 (i)

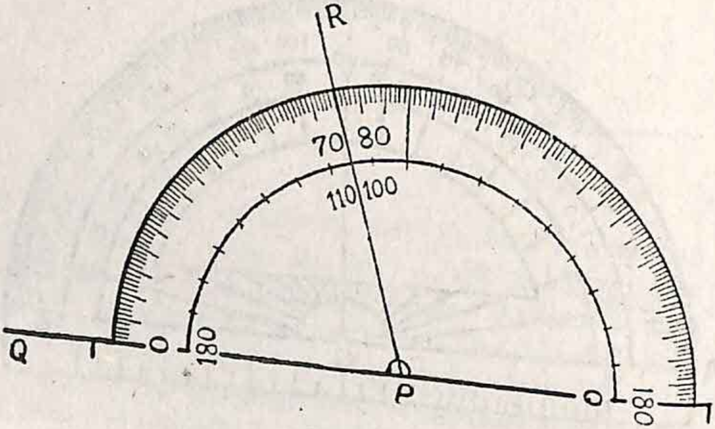


Fig-74 (ii)

Suppose we want to measure $\angle BAC$ [see fig. 74 (i)]. First you extend the sides of the angle if required, so that the sides go beyond the protractor where it is placed over the angle.

Next, set the protractor so that the vertex A of the angle coincides with the point O and one side of the angle coincides with its horizontal edge.

In fig. 74 (i), AB coincides with the horizontal segment lying on the right side of O. Now you observe at what mark of the protractor the other side of the angle meets. Determine the number of divisions on the protractor from O mark on the right side up to the mark located earlier. The number of division thus obtained if expressed in degrees, gives the measure of the angle BAC. Remember that the symbol $\angle BAC$ is used to express both the angle BAC and the measure of the angle BAC.

In fig. 74 (ii), you see that the arm PQ of the angle QPR coincides with left portion of the horizontal edge of the protractor. Now you have to measure the division starting from O on the left side. Verify the the measure of the angle QPR is 75°.

While measuring an angle, if one side of the angle does not meet a particular mark, note the nearest mark close to this and thus measure the angle.

Now you will see how an angle of given measure can be drawn with the help of a protractor.

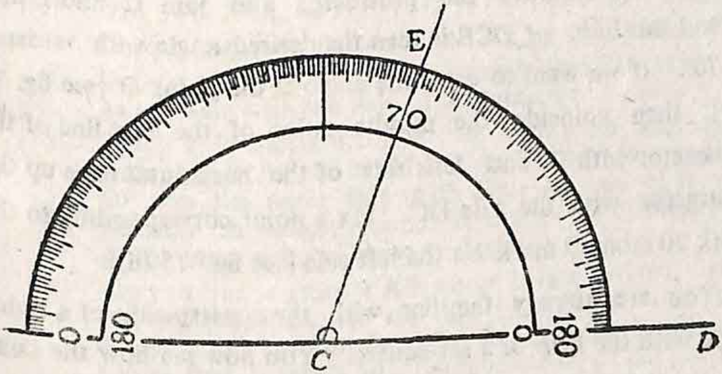


Fig-75 (i)

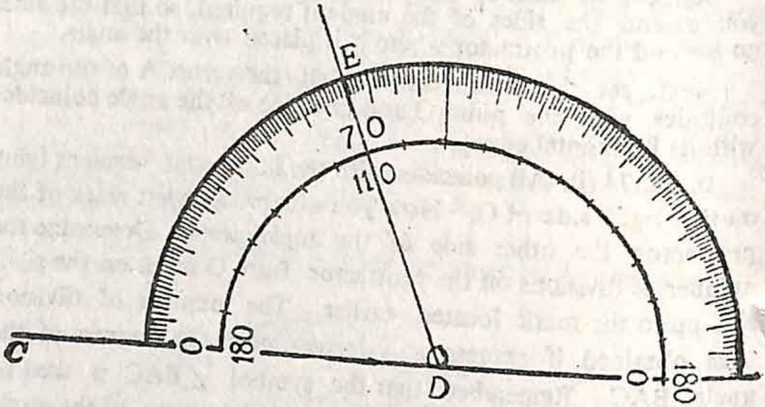


Fig-75 (ii)

Take a segment. Suppose we want to draw an angle of 70° at a point, say C on the segment. Place the protractor in such a way that the mid-point O coincides with the point C. Now move the protractor so that part of horizontal edge on the right coincides with CD [see fig. 75 (i)]. Starting from O mark the point which indicates the division 70. Let this point be E. Remove the protractor and join C and E and extend this line. ∠DCE is then the desired angle with measure of 70°. If we want to draw the angle at the point D [see fig. 75 (ii)], then coincide the middle point of the base line of the protractor with D and left side of the horizontal edge up the protractor with the side DC. Fix a point corresponding to the mark 70 from O mark on the left side [see fig. 75 (ii)].

You are already familiar with the construction of a right-angle with the help of a set-square. You now see how the same construction can be made by a protractor.

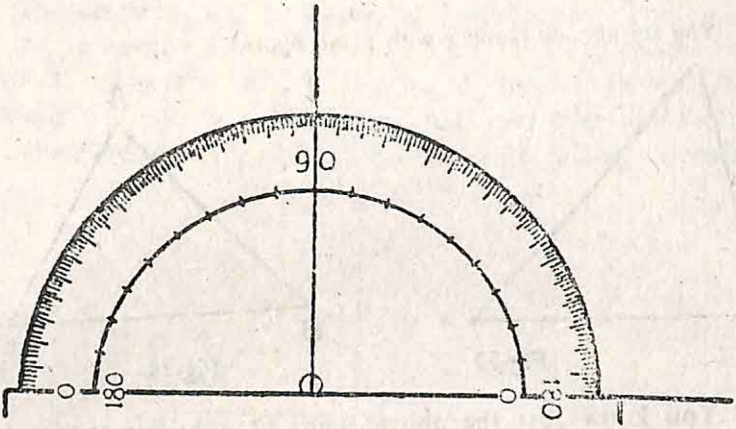


Fig-76

Measure an angle of 90° with the help of protractor by the method discussed earlier. A right-angle is thus obtained (fig. 76).

Exercise—6

1. Draw five angles and give their names. Measure these angles by a protractor.
2. Determine which of the following angles are acute and which are obtuse :
   55°, 110°, 70°, 45°, 170°, 180°, 90°, 42°, 113°.
3- Draw the following angles with the help of a protractor :
   91°, 52°, 135°, 90°, 73°, 15°, 161°, 35°, 68°, 105°.
4. Draw a perpendicular on a line AB by set-squares.
5. Draw a perpendicular on the straight line PQ taking it as one side with the help of a protractor.
6. Draw a line segment LN. At L draw an angle of 30° with LN as one of its sides. At N draw an angle of 60° on the same side with NL as one of its sides. Measure the angle formed at the point of intersection of the other sides of these angles.
7. Draw a line segmenet AB whose length is 6 cm. At A draw a perpendicular on AB. At B draw an angle BAC whose measure is 135°. Produce the line segment CD so that it meets the perpendicular at D. Measure the angle ADB.

10. Triangle

You are already familiar with plane figures.

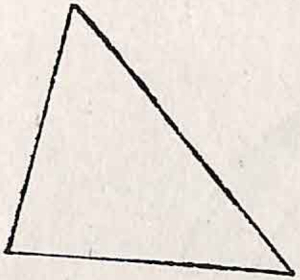


Fig-77

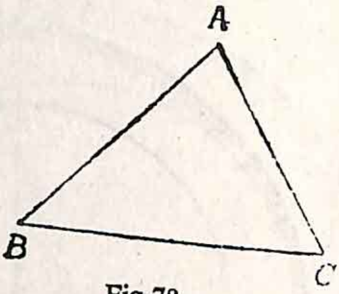


Fig-78

You know that the above figure-77 is known as triangle. Let the triangle be named as ABC (see fig. 78). The triangle in fig. 78 is called the triangle ABC or $\triangle ABC$. Segments AB, BC and CA are called sides of the triangle ABC. Sides AB and BC meet at B, sides BC and AC meet at C and sides AB and AC meet at A. These meeting points A, B, C are called vertices of the triangle ABC. $\angle ABC$, $\angle BCA$, $\angle CAB$ are known as three angles of the triangle ABC.

So, a triangle has three sides, three angles and three vertices.

11. Quadrilateral

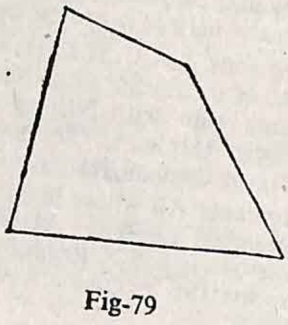


Fig-79

The above figure is known as **quadrilateral**. The name of the quadrilateral is ABCD. A, B, C, D are called its vertices. Segments AB, BC, CD and DA are sides of the quadrilateral. So, a quardrileteral has four sides, four angles and four vertices (fig. 79). Line segments joining opposite vertices are called its **diagonals**. AC and BD arc diagonals of the quadrilateral ABCD (see fig 80).

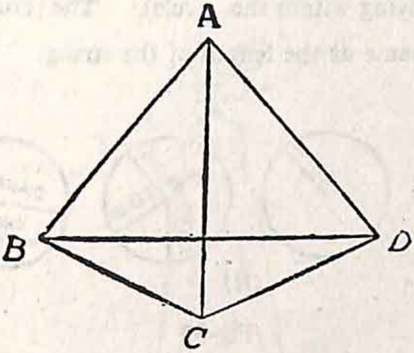


Fig-80

12. Circle

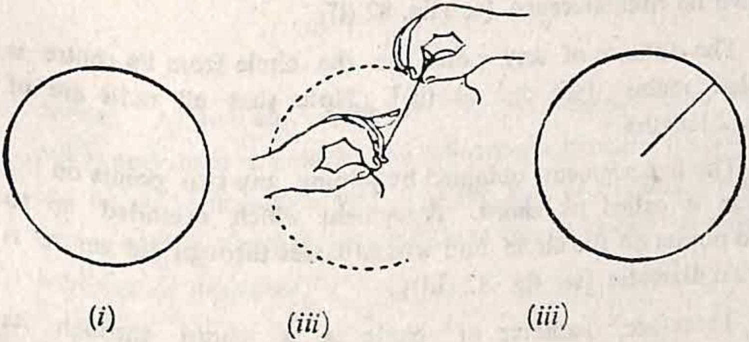
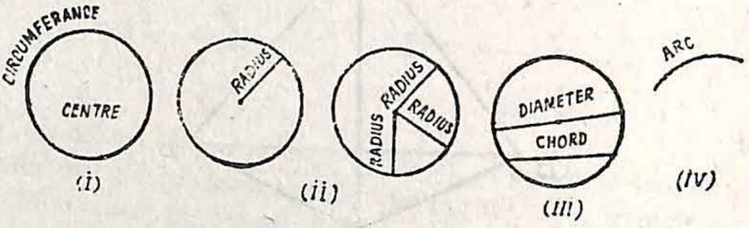

(*i*) (*iii*) (*iii*)

Fig-81

You are already familiar with circle. Many of you have constructed or have seen construction of a circle in play ground. If you keep one end of a string fixed and other end be resolved completely without making it loose, a circle is obtained, while moving the other end of the string, note that the fixed end is not displaced. So circle is a figure in which points are all situated at a constant distance from the fixed point (which is lying within the circle), The constant distance is found to be same as the length of the string.



(i) (ii) (III) (IV)

Fig-82

The point within the circle from which distances of all points on the circle are equal, is called **centre of the circle** [see fig. 82, (i)].

The length of the curved segment which make the circle is called its **circumference** [se e fig. 82 (i)].

The distance of any point on the circle from its centre is called **radius** [see fig. 82 (ii)]. Note that all radii are of equal lengths.

The line segment obtained by joining any two points on the circle is called its **chord.** A segment which extended up to two points on the circle and which passes through its centre is called **diameter** [see fig. 82 (iii)] .

Therefore, diameter of circle is a chord through its centre. Note that the length of a diameter is twice that of a radius.

A part of the circumference of a circle is known as arc [see fig. 82 (iv)].

13. Construction of a circle and arc of a circle

The instrument used for construction of a circle is known as compass (see fig. 83). The compass has two ārms like divider. At one end of the arm there is a pointer and at the other end there is a device for adjusting a pencil. Adjustment can be made in such a way that the lengths of both the arms remain same. The gap between these two arms can be suitably increased or decreased.

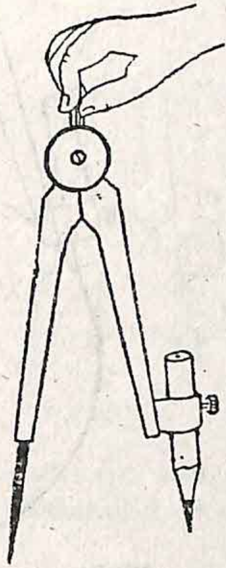


Fig-83

Construction of a circle

Take a point P and a line segment AB. We shall draw a circle with P as its centre and with the length of the segment as radius.

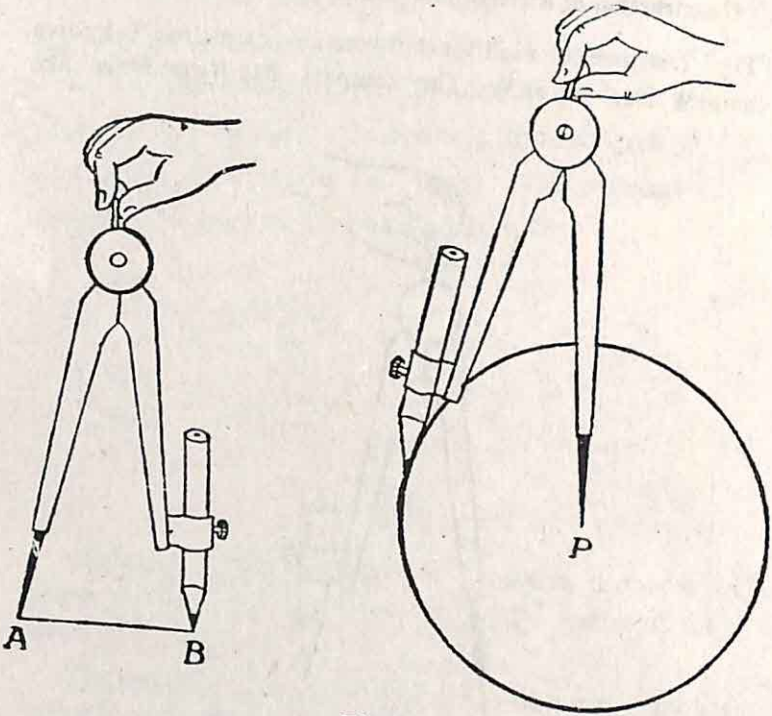


Fig-84

Place the pointer at the point A and the end of the pencil at the other point (see fig. 84). Now remove the compass and place the pointer at the point **P**. Then rotate the compass keeping the pointer fixed at **P** so the clear trace of the pencil ts obtained. Make one complete rotation in this manner- A circle is then drawn on the paper. While rotating the compass you should be very careful so that the distance between the pointer and the end of the pencil is not altered. Observe that the point where the pointer is placed is the center of the circle and radius of the circle is the distance between the end point of the pointer and end point of the adjusted pencil in the compass. To draw an arc of a circle,

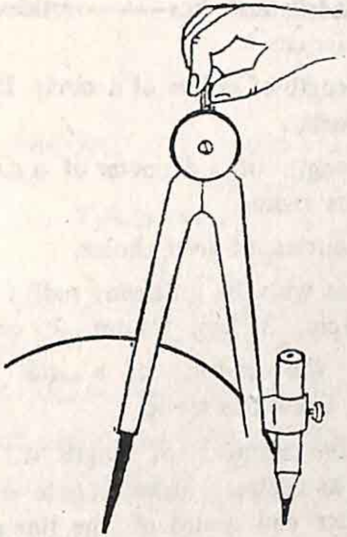


Fig-85

proceed as before but will not apply the full rotation. The figure thus drawn is the desired arc (see fig. 85),

Exercise—7

1. Construct three triangles and four quadrilaterals. Name these figures.
2. Construct a triangle. Name it and write down the names of their sides and vertices.
3. Construct a quadrilateral and name it. Write down the names of its four vertices and two diagonals.
4. Draw a circle and indicate on it the following : centre, radius, diameter, chord. Show them in the figure.
5. Fill up the following blanks :
   (a) A triangle has——vertices——sides.

(b) A quadrilateral has——— vertices; ——— sides, and ——— diagonals.

(c) The length of radius of a circle is——— the length of the diameter.

(d) The length of a diameter of a circle is——— the length of its radius.

6. Draw five circles of your choice.

7. Draw circles with the following radii :
4 cm, 2·5 cm, 3·2 cm, 5·1 cm, 3·9 cm.

8. What is the radius of a circle whose diameter is 8 cm ? Draw this circle.

9. Draw a line segment of length 4·2 cm. Taking one end point as centre, draw a circle with radius 2·2 cm. With other end point of the line segment as centre and radius 2 cm, draw another circle.

10. Draw a line segment of length 6 cm. Taking one of its end point as centre, draw a circle of radius 3 cm and another circle of radius 2 cm with other end point as centre.

11. Take a line segment of length 4 cm and taking its. two end points as centres, draw two circles of radius 2·5 cms each.

12. At the end points of a line segment whose length is 4 cm, draw two circles each of radius 2 cm.

13. At the end points of a line segment of 4 cm, draw two circles at its two end poins as centres and with common radius 1·3 cm.

14. Draw two arcs of a circle of arbitrary radius.

15. Construct an angle of 120° with the help of a protractor. Taking vertex of the angle as centre, draw an arc of a circle with any radius so that the arc intersects the sides of the angle.

SECOND CHAPTER

Construction

You already know how to draw a segment, an angle, a circle and arc of a circle. You will now learn few more constructions.

At different steps of these constructions, separate diagrams are given so that you can understand them clearly ; but you will draw all the steps in the same figure.

1. To bisect a given line segment

First step :

Take a line segment AB.

[see fig. 86 (i)].

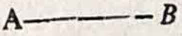


Fig-86 (i)

Second step :

Draw a circle with B as centre and AB as radius [see fig-86 (ii)].

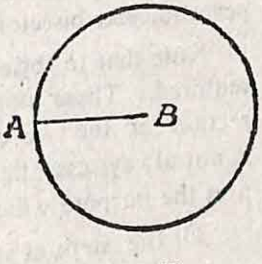


Fig-86 (ii)

Third step :

Taking A as centre and AB as radius draw another circle. Two circles now intersect each other [see fig. 86 (ii)] at P and Q.

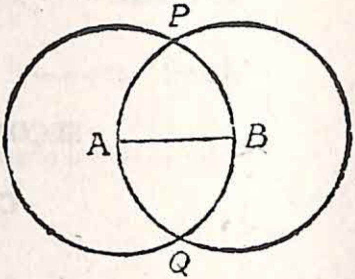


Fig-86 (iii)

Fourth step :

Join P and Q. The straight line PQ intersects the line segment AB at R. As a result the line segment AB is bisected at the point R. The straigh line PQ is called the bisector of the line segment AB.

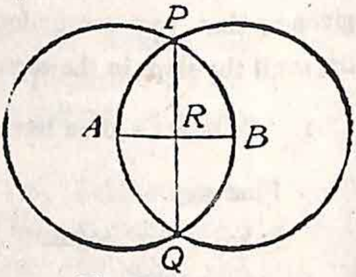


Fig-86 (iv)

With help of a protractor, measure the angles ∠ARP and ∠PRB. You will find that each angle is 90°. That is, straight line PQ is perpendicular to AB. Therefore, line PQ is the perpendicular bisector of the line segment AB.

Note that to bisect a line segment, two points P and Q are required. These points are found to be the points of intersection of the two circles. Hence, construction of two circles is not always essential. If one constructs only two intersecting arcs the purpose will be served.

All the steps of such construction are gived below :

First step :

Take a line segment AB.

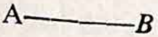


Fig-87 (i)

Second step :

Draw an arc of a circle with B as centre and AB as radius.

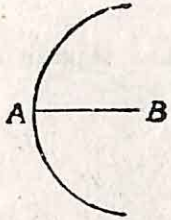


Fig-87 (ii)

Third step :

With A as centre and AB as radius draw another arc.

Two arcs intersect at the points P and Q,

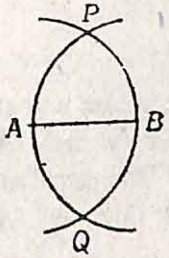


Fig-87 (iii)

Fourth step :

Join P and Q. The straight line PQ intersects the line segment AB at R.

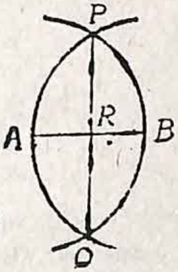


Fig-87 (iv)

Therefore, the line segment AB is bisected at the point R.

If you are now able to draw small arcs then you can proceed as follows:

First step :

Take the line segment AB [fig.-88 (i)].

Fig-88 (i)

Second step :

With B as centre and the length of the line segment AB as radius draw the small arcs lying on both sides of AB [fig. 88 (ii)]

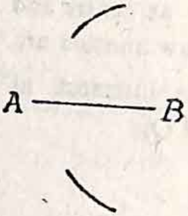


Fig-88(ii)

Third step :

Similarly, draw two other small arcs with A as centre and AB as radius so that two arcs on one side intersect at P and on the other side interesect at Q [fig. 88 (iii)].

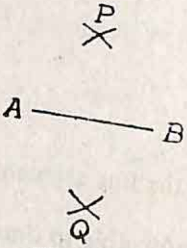


Fig-88 (iii)

Fourth step :

–Join P and Q. The straight line PQ intersects the line segment AB at the point R.

Thus the line segment AB is bisected at the point R.

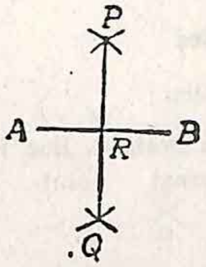


Fig-88 (iv)

[Note : In above constrnctions, the radius of the arc may be less than the length of the line segment. As a matter of fact the constructions could be made by taking the radius more than half of AB.]

Measure the line segment AR by a divider and placing the divider on the line segment RB, verify whether two line segment are equal or not.

Exercise—8

1. Draw 6 cm long line segment with the help of a ruler. Bisect this line segment.
2. Divide the line segment LM into four equal parts.
3. Divide the line segment LD into eight equal parts.
4. Take a line segment PQ. Draw a circle with PQ as diameter.
5. Take a triangle ABC. Construct the perpendicular bisectors of its three sides.

5. To draw a perpendicular on a given straight line from a point lying outside the straight line.

First method

First steep :

AB is a straight line, P. is an external point [fig 89. (i)].

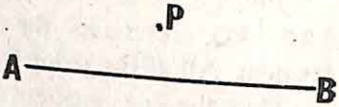


fig-89 (i)

Second step :

The point P lies, on one side of AB. On the otherside of AB, take a point Q. With P as centre and PQ as radius, draw an arc which intersects AB at the points C and D [fig. 89 (ii)].

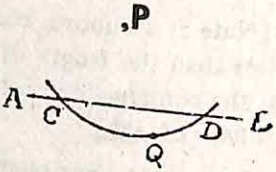


Fig-89 (ii)

Third step :

On the opposite side of P, draw two small arcs with C and D as centres and of redius more than half of the length of line segment CD. Let these two arcs intersect at the point R [fig. 89 (iii)].

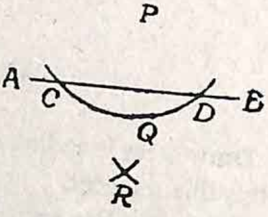


Fig-89 (iii)

Fourth step :

Join P and R. The straight line PR intersects AB at O. Then the straight line PR is the desired perpendicular drawn on the straight line AB from the external point P.

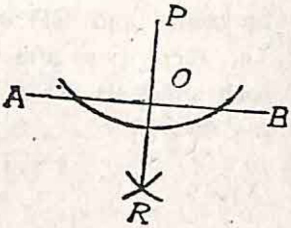


Fig-89 (iv)

[In each case measure the angles POB and POA with the help of a protractor and verify whether each of them is equal to 90° or not.]

Second Method

First step :

AB is a straight line and P is an external point relative to it [fig, 90 (i)].

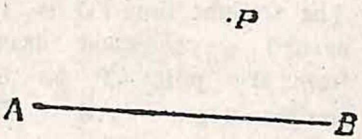


Fig-90 (i)

Second step :

Take any two points C and D on the straight line AB [fig, 90 (iii].

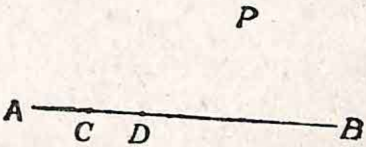


Fig-90 (ii)

Third Step :

Draw an arc with C as centre and CP as radius [fig, 90 (iii)].

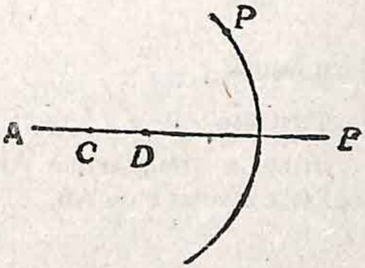
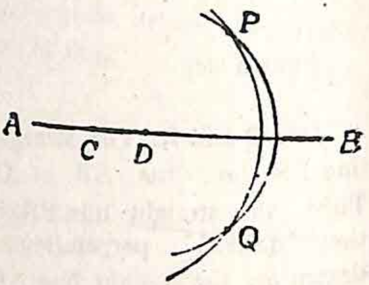


Fig-90 (iii)

Fourth step :

Draw another arc with D as centre and DP as radius. Let these two arcs intersect each other also at the point Q [fig. 90 (iv)].



Fifth step :

Join P and Q and produce. The straight line PQ is the desired perpendicular drawn from the point P on the straight line AB [fig. 90 (v)].

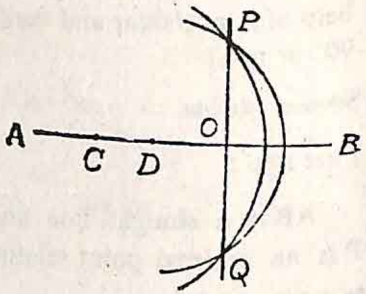


Fig-90 (v)

3. To draw a perpendicular at a point on a given straight line.

First method

First step :

Draw a straight line AB and take a point P on AB.

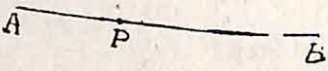


Fig-91 (i)

Second step :

With P as centre draw an arc with any radius so that it intersects the straight line AB at the points C and D. [fig. 92 (ii)].

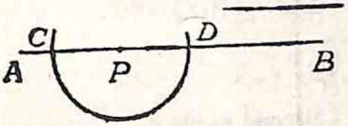


Fig-91 (ii)

Third step :

With D as centre and with radius more than the length of DP, draw an arc on any side of the straight line AB [fig. 91 (iii)].

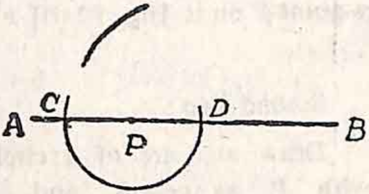


Fig-91 (iii)

Fourth step :

Taking C as centre with same radius (as in previous tep), draw an arc on the same side of AB. Let these two arcs intersect at the point Q [fig. 91 (iv)].

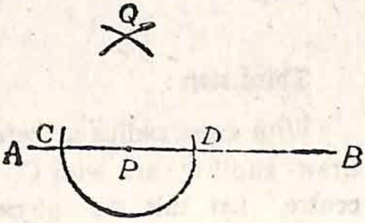


Fig-91 (iv)

Fifth step :

Join P and Q. The straight line PQ is then the required perpendicular at P on AB [fig. 91 (v)].

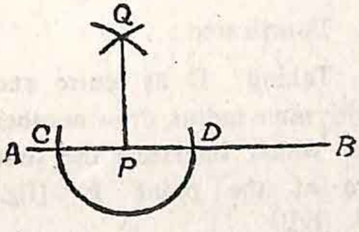


Fig-91 (v)

With the help of protractor, verify that the ∠APQ=90° and ∠BPQ=90°.

Second method

First step :

Take a straight line AB and a point P on it [fig. 92 (i)],

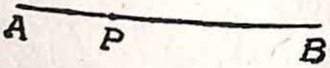


Fig-92 (i)

Second step :

Draw an arc of a circle with P as centre and an arbitrary radius so that it intersects AB at C [fig. 92 (ii)].

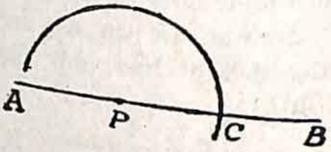


Fig-92 (ii)

Third step :

With same radius as before draw another arc with C as centre. Let this arc intersect the previous arc at the poiut D [fig. 92 (iii)].

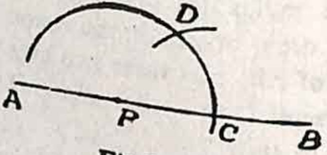


Fig-92 (iii)

Fourth step :

Taking D as centre and with same radius, draw another arc which intersects the first arc at the point E [fig. 93. (iv)].

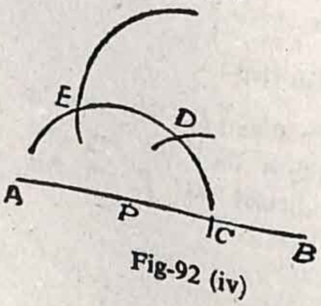


Fig-92 (iv)

Fifth step :

Again, take E as centre with same radius and draw an are which intersects the previous are at the point Q [fig. 92 (v)].

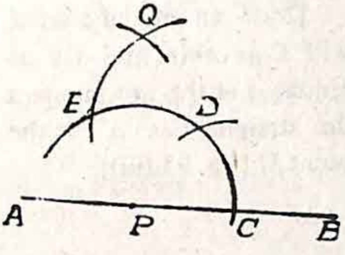


Fig-92 (v)

Sixth step :

Join P and Q. Straight line PQ is thus the desired perpendicular drawn on the straight line AB at the point P.

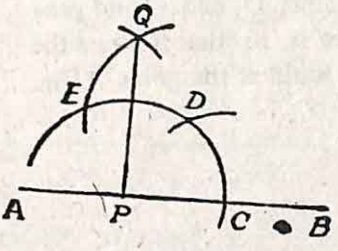


Fig-93 (vi)

Third method

First step :

Take a point P on a straight line AB. [fig. 93 (i)].

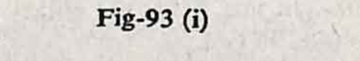


Fig-93 (i)

Second step :

Take a point C external to the straight line AB [fig. 93 (ii)].

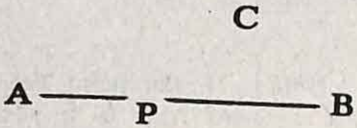


Fig-93 (ii)

Third step :

Draw an arc of a circle with C as centre and CP as radius. Let this arc intersect the straight line AB at the point D [fig. 93 (iii)].

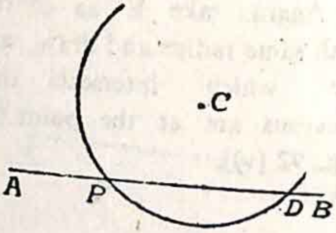


Fig-93 (iii)

Fourth step :

Join D and C and produce it so that it meets the arc again at the point Q [fig. 93 (iv)].

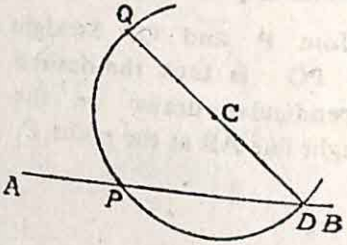


Fig-93 (iv)

Fifth step :

Join PQ and produce. PQ is thus the required perpendicular drawn on the straight line AB at the point P [fig. 93 (v)].

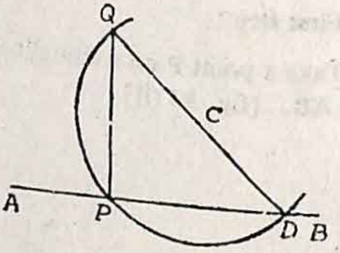


Fig-93 (v)

Note : If the point becomes one of the end points of the line segment AB, it is advantageous to apply the second or third method.

Exercise—9

1, From an external point A, draw a perpendicular on the straight line PQ.

2. At a point B on the straight line LM, draw a perpendicular on LM.

3. Take a line segment CD whose length is 6 cm. At the point C, draw a perpendicular on CD.

4. Take a straight line AB. Cut off AC=4 cm and CD=5 cm from the straight line AB, such that the points C and D lie on opposite side of the point A. Draw perpendiculars on the straight line AB at points C and D.

5. Take a straight line AB. At a point C on AB, draw a perpendicular CD on AB. Again, take a point E on CD and at E draw a perpendicular on CD.

6. Take a triangle ABC. From the vertices A, B and C, draw three perpendsculars on BC, CA and AB respectively.

7. Take a quadrilateral ABCD. Draw perpendiculars on the diagonal AC from the points B and D.

8. Take a triangle PQR. Draw perpendiculars PM and QN on PQ at points P and Q respectively. Draw perpendicular on PM from the point R. Measure the angle that the last perpendicular makes with QN.

4. To bisect a given angle

First step :

Take an angle ABC [fig. 94 (i)].

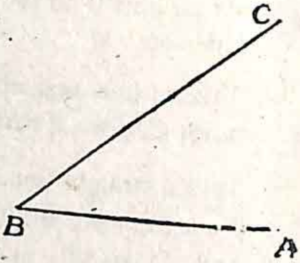


Fig-94 (i)

Second step :

Draw an arc of a circle with B as centre and with an arbitrary radius so that it intersects the sides BA and BC of the angle at the points P and Q respectively [fig. 94 (ii)].

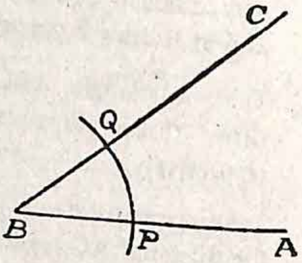


Fig-94 (ii)

Third step :

With P as centre and radius more than half of PQ, draw an arc which lies on the opposite side of B [fig. 94 (iii)].

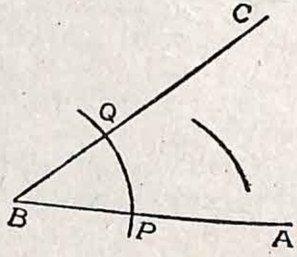


Fig-94 (iii)

Fourth step :

With same radius draw another arc with Q as centre so that this arc intersects the previous one at the point D [fig. 94 (iv)].

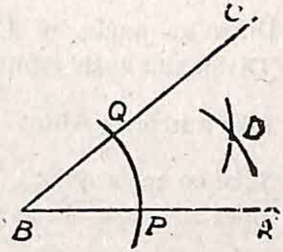


Fig-94 (iv)

Fifth step :

Join B and D The straight line BD then bisects the ∠ABC. Therefore, the straight line BD is the bisector of the ∠ABC [fig. 94 (v)].

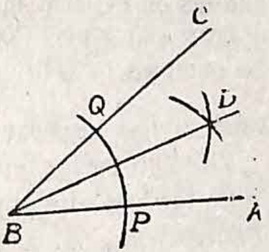


Fig-94 (v)

By protractor, measure ∠CBD and ∠DBA and verify that their measures are same.

Exercise—10

1. Draw an angle of 56° with the help of a protractor. Bisecet this angle.
2. At the point B on the straight line AB, draw a perpendicular BC. Bisect the ∠ABC.

3. Draw an angle of 120° with the help of a protractor Divide this angle into four equal parts.

4. Take a triangle ABC. Bisect all its three angles.

5. Take an angle PQR. Bisect this angle. Produce RO upto S. Bisect the angle SQP.

6. Take a quadrilateral ABCD. Bisect its angles.

7. Draw a line segment PQ. Draw perpendiculars PR and QS on PQ at points P and Q respectively. Bisect ∠QPR and ∠PQS. Measure the angles of the triangle So obtained.

9. Draw a line segment XY of length 6 cm. Draw ∠YXZ at the point X and ∠XYT at the point Y with the help of a protractor such that their respective measure are 60° and 120°. Bisect these angles. Measure the sides and angles of of the triangle so formed.

B. L. / Math. VI (E) / 1996

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